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SPACEBORNE CENTRIFUGAL RELAYS
FOR SPACECRAFT PROPULSION

BY

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ABSTRACT

Acceleration using centrifugal relays is a recently discovered method for the acceleration of spaceborne payloads to high velocity at high thrust. Centrifugal relays are moving rotors which progressively accelerate reaction mass to higher velocities. One important engineering problem consists of accurately tracking the position of the projectiles and rotors and guiding each projectile exactly onto the appropriate guide tracks on each rotor. The topics of this research are the system kinematics and dynamics and the computerized guidance system which will allow the projectile to approach each rotor with exact timing with respect to the rotor rotation period and with very small errors in lateral positions. Kinematics studies include analysis of rotor and projectile positions versus time and projectile/rotor interactions. Guidance studies include a detailed description of the tracking mechanism (interrupt of optical beams) and the aiming mechanism (electromagnetic focussing) including the design of electromagnetic deflection coils and the switching circuitry.

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Chapter 1

INTRODUCTION

Three billion years ago, organic life began to develop rapidly on spaceship Earth. It was powered by energy of nuclear origin transmitted from the sun in the form of light. Large amounts of that solar energy were then stored in the form of coal, oil, and gas following the death of trees and other living organisms. More recently, man appeared on the spaceship. Until about 200 years ago or so, men lived in small separate groups. Then they began to integrate their activities into modern societies making the need for energy bigger and bigger until they reached a state where those easy-to-get hydrocarbons are almost gone. The days of cheap fossil energy are almost over [1]. Nuclear fission energy cannot be more than a transitional scheme since the fuel supply is limited and the question of nuclear waste has not been solved. People are now turning to solar energy and thinking about going to outer space where it can be easily harvested. This idea first appeared in 1968 when Peter Glaser proposed a project for orbital space power satellites [2]. In the mid-seventies, Gerard O'Neill developed further the concept of space industrialization [3, 4], and by the late-seventies, the idea had gathered momentum and consideration among industry and government

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in the United States, Europe, and the former USSR [1].

As we desperately search for new sources of energy, space industrialization, with heavy production of aluminium, iron, silicon, and other energy consuming products, appears as an encouraging way out of the current crisis [1]. Recent years have seen encouraging progress in studies of the transport, smelting, and use of nonterrestrial resources for large-scale space industrialization. A promising approach to transporting such resources is the launch from the moon of unprocessed lunar ore. The ore packages would be captured and processed in space [5]. Metallic ore can be obtained not only from the moon surface, but also from asteroids and it can be processed in high Earth orbit as suggested by O'Neill in 1976 [6]. The products can then be used for the construction of space structures or sent back to the surface of the Earth casted into re-entry bodies [1]. It appears though that the utilization of lunar materials will almost certainly be a near term necessity while asteroidal resources will probably not be tapped for at least another ten years according to the most optimistic estimates [7]. The need for fully developed industrial schemes both in space and on the surface of the moon, and for sophisticated propulsion systems is then explicit.

1.1 HISTORICAL CONTEXT

Current launch vehicle systems such as the Space Shuttle, involve launch costs on the order of $1000/kg of payload [8]. Before the large-scale development of space resources and energy can succeed, launch costs must be reduced. Launch costs for liquid-fuel rockets cannot be lower than the cost of the fuel, which is about $3 /kg for liquid hydrogen [9]. This limits the cost of launching to a minimum of $50/kg of payload, not considering vehicle development and launch operation costs [8]. This cost may be too high for cheap

space industrialization. Various schemes have been proposed for low-cost launch. L. W. Jones et al. proposed a laser propulsion system [10], where a ground-based laser would provide the external power. A space elevator with electrical power distributed along its length called "The Orbital Tower" was suggested by J. Pearson [11]. Y. Artsutanov et al. [12] and H. Moravec [13] proposed a rotating tether to pick up large payloads from the ground or upper atmosphere. A launch loop or "Orbital Ring," with rapidly moving internal components to support it against gravity was discussed by P. M. Birch [14] and K. Lofstrom [15].

All these different schemes are attractive theoretically, but they require either great improvements in materials or lasers, enormous masses in orbit, or large, dynamically stabilized masses overhead [8]. A compromise system putting most of the energy requirements on the ground and requiring minimum mass in orbit has been suggested by J. Pearson [8] in the late-eighties. It consists of a relatively small electromagnetic gun on the ground with a relatively low-mass rotating tether in orbit to catch the projectile. This system requires high launch accelerations and is then limited to unmanned payloads.

Another concept that got the attention of many scientists is the principle of solar sails. They have been the subject of several studies on external momentum sources for accelerating objects in space. The momentum is obtained when photons bounce off a reflector [16]. Lasers have been suggested as an alternative to the sun [17], but the problem is that photons are inefficient for this purpose and lead to low momentum/energy ratio. The photon is a poor choice of momentum transfer. An additional problem is the quantum mechanics limitations. It is difficult to collimate a beam adequately. Therefore, solar sails are limited to very low thrust and simple laser-driven sails are impractical [18].

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To overcome the limitations of solar and laser-driven sails, the use of macroscopic objects to transfer momentum from a remote source appeared more promising. A kinematic study to this approach has been worked out [19] and the requirements for maintaining a collimated stream of projectiles is achievable [20]. The unsolved problem was the production of a suitable stream of high velocity projectiles. Two methods of launching such projectiles from extraterrestrial platforms were investigated. The first one is the linear induction motor [21, 22]. Unfortunately, the required length of a linear induction motor increases with the square of the launch velocity, making the high-velocity launcher very cumbersome. The second method consists of using centrifugal launchers.

1.2 CENTRIFUGAL LAUNCHERS

As an alternative to both chemical and elctromagnetic launchers, centrifugal projectile launchers were proposed to provide reliable, efficient, compact systems that can accelerate projectiles to $2 - 3$ km/s with energies of 10^5 to 10^6 J without chemical explosives [23]. The direct conversion of rotational mechanical energy into transational mechanical energy with no intermediate stores or transfers, makes the potential efficiency in repetitive operation higher than any other system. Otherwise, a homopolar generator/rail gun system must convert mechanical energy to electrical, store the electrical energy inductively, convert the electrical energy to plasma heat and kinetic energy, and finally transfer the plasma kinetic energy to projectile kinetic energy [23]. The compactness is limited mainly by the yield strength, which is the ultimate limitation to compactness of any solid projectile launcher.

The use of centrifugal launchers in space had been suggested for lifting lunar material resources off the moon in the late-seventies [21, 5], but rotors

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capable of the required 2.2 km/s launch had not been developed yet, so long linear induction launchers were chosen as the primary launch option [24].

During the fall and winter of 1976, H. Kolm and G. O'Neill made a device that would operate at high acceleration (100 g) necessary for a mass driver to be used in space. The model served to educate people on the principle of mass driver design and also demonstrated that such acceleration could be achieved with fairly simple machinery [25].

The performance of centrifugal launchers became more known as further studies were conducted and the method is now widely used for refuelling a plasma in toroidal magnetic confinement devices such as tokamaks and stellarators/heliotrons [26]. Rotors injecting frozen hydrogen pellets into hot plasmas were tested at rotor tip speed of 1 km/s. As the fragile hydrogen pellets tend to break up at somewhat lower tip velocities, there seemed to be little incentive to develop higher tip speeds theoretically allowable by mechanical stress limitations [24]. The proposed lunar application to centrifugal launchers is for launching large sacks of raw dirt and the existing type of high-velocity centrifugal launcher is best suited to material refined into small balls. Therefore, these centrifugal launchers are not feasible for lunar application but they may be useful for propelling spacecraft payloads [24].

Unfortunately, stress considerations limit simple centrifugal launchers to a launch velocity not greatly exceeding the exhaust velocity of chemical rockets [18]. Recently, however, a method has been proposed for substantially augmenting the projectile velocities using centrifugal launchers [24].

1.3   CENTRIFUGAL RELAYS METHOD

The basic idea starts with an existing centrifugal launcher [27] as illustrated in Figure - 1 [24]. The mass driver is anchored on a large body and used

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DRIVE SHAFT

GUIDE TUBE

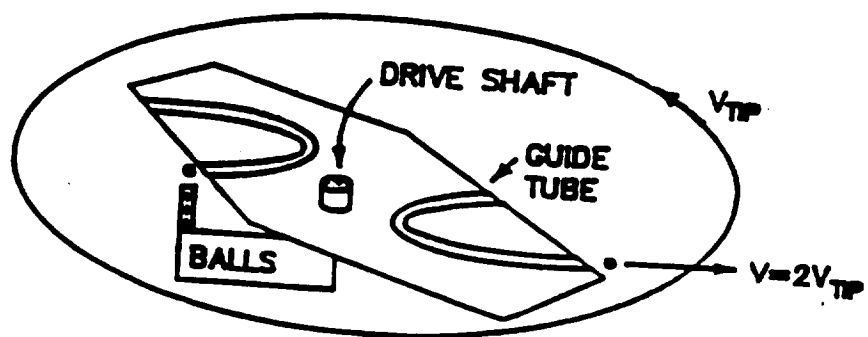BALLS

V_{TIP}

$V = 2V_{TIP}$

Figure - 1. Centrifugal launcher.

to throw small masses towards a curved track or guide tube on the payload. Momentum may be transferred by stopping these masses at the payload, or more efficiently, by reflecting them back to the launcher [24]. These two options are summarized in Figure - 2 [24].

To increase the velocity of the projectiles incident on the payload, a centrifugal relay is inserted between the launcher and the payload as illustrated in Figure - 3 [24]. The requirements on this relay are: its rotational energy must be maintained so that it accelerates incoming projectiles to a higher speed when passing them on to the payload; and it must be continuously accelerated so that its velocity is intermediate between the first launcher and the payload. This method can be extended by inserting two or more centrifugal relays between the primary launcher and the payload [18]. This makes this recently discovered method using centrifugal relays for the acceleration of spaceborne payload to high velocity at high thrust, very attractive. It has unique advantages over chemical rockets (low velocity - high thrust), and other advanced methods such as plasma thrusters and solar sails (high velocity - low thrust) [18].

1.4 OBJECTIVE

The purpose of this work is to explore the use of centrifugal relays for spacecraft propulsion in sufficient detail to assess the potential of this method for orbit transfers requiring large velocity increments (Δv's up to 10 km/s), and for the capture into Earth orbit of suborbital payloads. The investigation of this potentially viable novel spaceborne propulsion concept is a joint project between the Aeronautical and Astronautical Engineering department, and the Nuclear Engineering department at the University of Illinois. This new initiative in advanced propulsion systems is funded by NASA.

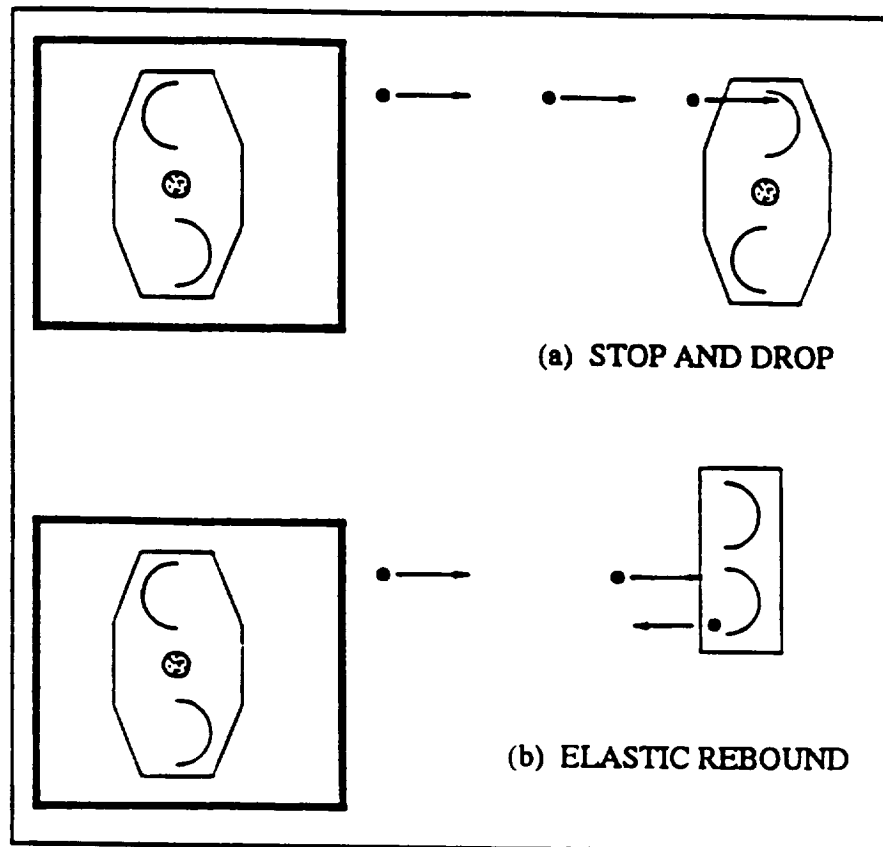(a) STOP AND DROP

(b) ELASTIC REBOUND

Figure - 2. Ball-payload interation for
a) stopping and
b) reflection.

2 KM/S

2.9 KM/S

PRIMARY
$V_2 = 0$

RELAY
$V_1 = 0.9$ KM/S

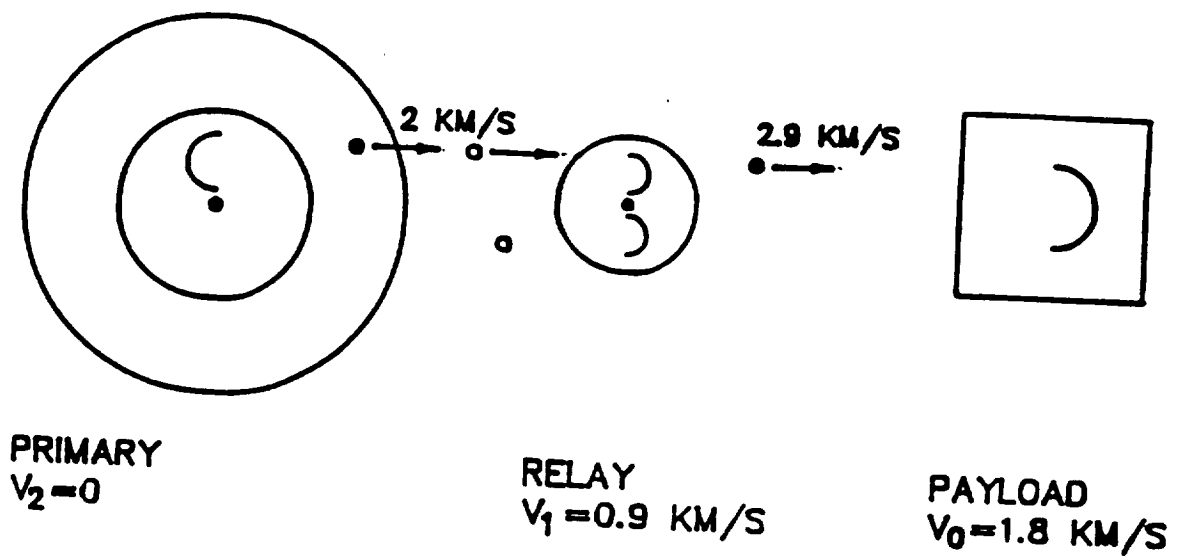**PAYLOAD**
$V_0 = 1.8$ KM/S

Figure - 3. Centrifugal relay method.

1.5 DEFINITION OF THE PROBLEM

For the purpose of this thesis, the goal is to assess the potential of centrifugal relays for a specific type of mission. The research can be divided into four areas: kinematics, mechanical design, guidance, and systems. We propose to investigate two aspects of this project and identify all the requirements for the kinematics and the guidance system. We will then identify the concepts of the solutions to each problem and we will define the components of each part.

We will start in *Chapter 2* by giving an overview of the launch system as a whole, followed by a description of the kinematics involved to understand how momentum is transferred. We will also discuss design considerations for the rotors. We will then give in *Chapter 3* a detailed description of the launch of one ball. Then, we will add to it the catching part in *Chapter 4*. We will also consider in this chapter the trajectory of the ball on the catching rotor and investigate a delay circuit that will insure that the ball flies off this rotor in the desired direction (which is, in this case, back to the launcher). After this one ball, one launcher, and one relay (payload in this case) problem is solved, we can consider the case of launching two or more balls. Then we can study the problem of additional tracks on the relay. The purpose would be to launch balls in the opposite direction and insure conservation of the angular velocity of the rotor. This generalization process of launching n balls is described in *Chapter 5*. We also develop in this chapter a program that keeps track of the position of the relay as the balls are launched. It compiles all the data necessary for the visualization of the process, such as the time at which a ball is released near the center of the primary launcher, the time at which it exits the launcher, the time at which it is caught, sent back to the

launcher, and at what speed the relay is moving. Finally, this rotor is to be considered as a launcher and the sequence repeats itself. The final step would be to write a general code that treats arbitrary numbers of relays taking into account the externally imposed gravitational potential appropriate to the mission type, but this is not in the scope of this thesis. We are limiting ourselves to the complete study of a one relay case. For guidance purposes, we will be considering guide barrels as a first step in velocity trimming. Electromagnetic focussing has been identified as the aiming mechanism. We will be giving in *Chapter 6* a detailed description of the tracking mechanism (interrupt of optical beams) and the switching circuitry for the deflection coils and all the electronics involved in the guidance system. The final step would be to consider the system as a whole for optimization of mission profiles. One could study the Mars mission. One could estimate the performance and the cost of such system, and this last part is again suggested as a continuation to this work and not part of it. We will be giving in *Chapter 7* our concluding remarks. We will summarize the results and discuss future work that we suggest necessary before the completion of this project as a whole.

Chapter 2

KINEMATICS AND DYNAMICS OF THE CENTRIFUGAL RELAY DESIGN

Understanding the kinematics of the centrifugal relay system is of great importance to this project. Before we start our kinematic study, we will give a brief overview of the whole launch system in order for the reader to get a clear picture of the concept. We will also discuss design considerations for the centrifugal launchers and come up with a conceptual design for the rotors.

2.1 OVERVIEW

The overall launch system is outlined in Figure - 4 [24]. For the first step of the launch process, a large flywheel is geared to a small primary centrifugal launcher. The power source could be nuclear, bioconversion, or physical conversion of solar energy [19]. Ferromagnetic ball bearings are released near the axis onto a curved track, and exit into a guide barrel. The spherical shape of the projectiles would minimize the influence of solar winds according to a study by Brian Von Herzen [28], although we are not concerned in this thesis

Conventional motor-driven
fly-wheel spun up with a
solar or nuclear electric system
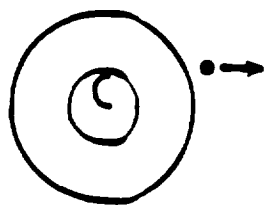
Primary

$V = 0$

Tracking and Aiming
Mechanism

Relay

$V_2 = V_0/2$

Payload

$V = V_0$

Figure - 4. Components of a launch system with one centrifugal relay.

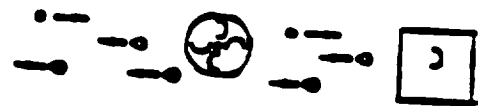with the external perturbations from gravitational fields and solar winds. For position and timing measurements, each ball interrupts at least two sets of horizontal and vertical laser beams directed upon fine-scale photodiode arrays. For aiming, sets of top/bottom and left/right coil pairs are energized to impart small lateral velocity increments. Solenoidal coils surrounding the flight path are used to exactly time the arrival of each ball at a target rotor 0.1–10 km downstream.

The stream of balls so launched first encounters a spinning relay rotor. In a spaceborne system, deflector shields would protect the relay against rare misaimed balls. The rotating balls land on a guide track with velocities matched to the track velocity so that they roll up the track. Present plans call for the relay rotors to be made of non-ferromagnetic composite materials. Coils imbedded near the rotor axis slow down the motion so that, when a ball rolls back down the track, it leaves going almost exactly forward or backward. Forward-going balls go on to impel the next relay or the payload assembly; backward-going balls are collected at the primary launcher. Additional tracks on each rotor assembly are used to process balls which land near axis to maintain nearly constant rotational energy and/or are being passed backward for eventual collection and reuse.

For a detailed description of centrifugal relays for spacecraft propulsion, the reader may refer to "Centrifugal Relays for Spacecraft Propulsion" by Clifford Singer and Richard Singer [24].

For the purpose of this thesis, we will be considering just one relay and the balls are only going backward to the primary launcher. The method is general and the results can be generalized easily for forward going balls impelling other relays or the payload. Before doing this, however, it is desirable to treat the one relay case in all details as completely as possible. We will

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start here by describing the kinematics involved in this project and show how the centrifugal launchers can be used as energy and momentum source for spaceborne propulsion systems.

2.2 KINEMATICS

The purpose of these kinematics calculations is the determination of momentum transfer. We will start by studying the case of a one ball launch and see how that ball transfers momentum to the payload. We will then generalize the results and write a recursive formula that gives the momentum gained by the payload as a function of the number of balls launched. This study will allow us to estimate the amount of initial stored energy required to launch a payload.

2.2.1 Momentum Transfer after the Launch of the First Ball

The approach here is similar to the one followed by Clifford Singer and Richard Singer [24]. The general linear momentum conservation equation can be written for this system as:

$$m\vec{v}_{in} + M\vec{V}_{in} = m\vec{v}_{out} + M\vec{V}_{out}, \qquad (1)$$

where m is the mass of the ball and M is the mass of the rotor catching the ball or, eventually the payload. $\vec{v}_{in}$ is the velocity that the ball has when it leaves the primary launcher and $\vec{v}_{out}$ is its velocity after it impacts the relay or payload, and rolls back off the track heading back toward the primary launcher. $\vec{V}_{in}$ is the relay/payload velocity before the launch of this first ball, and $\vec{V}_{out}$ is its velocity after this first launch is completed.

The energy conservation equation can be expressed in terms of the rota-

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tional energy E_{rot}, gained by the relay as:

$$E_{rot} = \underbrace{\left[\frac{1}{2}mv_{in}^2 + \frac{1}{2}MV_{in}^2\right]}_{\text{total initial kinetic energy}} - \underbrace{\left[\frac{1}{2}mv_{out}^2 + \frac{1}{2}MV_{out}^2\right]}_{\text{total final kinetic energy}} , \qquad (2)$$

or,

$$2E_{rot} = m(v_{in}^2 - v_{out}^2) + M(V_{in}^2 - V_{out}^2). \qquad (3)$$

Let us denote by $\Delta\vec{v}$ the decrease in ball velocity,

$$\Delta\vec{v} = \vec{v}_{in} - \vec{v}_{out}. \qquad (4)$$

Then, the momentum lost by the ball is $m\Delta\vec{v}$. It corresponds to the momentum gained by the rotor $M(\vec{V}_{out} - \vec{V}_{in})$. So, the momentum transfer can be written as:

$$M(\vec{V}_{out} - \vec{V}_{in}) = m\Delta\vec{v}. \qquad (5)$$

Equation (3) can be rewritten as:

$$2E_{rot} = m(\vec{v}_{in} - \vec{v}_{out})(\vec{v}_{in} + \vec{v}_{out}) + M(\vec{V}_{in} - \vec{V}_{out})(\vec{V}_{in} + \vec{V}_{out}),$$

and, using Equations (4, 5), it becomes:

$$2E_{rot} = m\Delta\vec{v}(\vec{v}_{in} + \vec{v}_{out}) - m\Delta\vec{v}(\vec{V}_{in} + \vec{V}_{out}).$$

Now, we can still substitute $(\vec{v}_{in} - \Delta\vec{v})$ for $\vec{v}_{out}$ from Equation (4) and, $(\vec{V}_{in} + \frac{m}{M}\Delta\vec{v})$ for $\vec{V}_{out}$ from Equation (5) in the equation above and get:

$$2E_{rot} = m\Delta\vec{v}\left[2(\vec{v}_{in} - \vec{V}_{in}) - \Delta\vec{v}(1 + \frac{m}{M})\right]. \qquad (6)$$

In the case of the payload, there is no rotational energy transfer $(E_{rot} = 0)$. Equation (6) can then be written as:

$$2(\vec{v}_{in} - \vec{V}_{in}) = \Delta\vec{v}(1 + \frac{m}{M}),$$

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or, using the definition of $\Delta \vec{v}$:

$$\vec{v}_{in} - \vec{v}_{out} = \frac{2(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}},$$

and finally,

$$\vec{v}_{out} = \frac{2\vec{V}_{in} + \vec{v}_{in}(-1 + \frac{m}{M})}{1 + \frac{m}{M}}. \tag{7}$$

We can also determine the increase in payload velocity as:

$$\vec{V}_{out} - \vec{V}_{in} = \frac{m}{M}\Delta\vec{v} = \frac{m}{M}(\vec{v}_{in} - \vec{v}_{out}).$$

Substituting for $\vec{v}_{out}$ from Equation (7), the increase in payload velocity becomes:

$$\vec{V}_{out} - \vec{V}_{in} = \frac{m}{M}\frac{2(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}}. \tag{8}$$

Now, if we go back to the case of a relay, the rotational energy is generally not zero. It can be obtained from Equation (6) provided $\Delta\vec{v}$ has been determined. In the next chapter, we will determine the final velocity of the ball, $\vec{v}_{out}$ after we analyze its interaction with the relay using Lagrangian mechanics. We have to mention here that our study will assume an infinite mass rotor. In practice, the rotor is about 10 kg and the ball is just a few grams. The ratio m/M is of order 10^{-4}, which justifies our assumption of infinite mass relay. If we were to take into account the finite mass of the relay, the Lagrangian of the system would be more complicated and the angular velocity of the relay would not be constant but would be an extra unknown to be determined. This would complicate the analysis. We will thus simplify the Lagrangian and study the interactions of the ball with an infinite mass rotor, keeping in mind that we have an error of the order of $(m/M)^2$ stepwise according to Equation (8), where the increase in velocity of the relay would be:

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$$\sim 2\frac{m}{M}(\vec{v}_{in} - \vec{V}_{in})\left(1 - \frac{m}{M}\right).$$

Knowing that $m/M \sim 10^{-4}$, we believe this is a quite good approximation and we will definitely consider in the next chapter an infinite mass rotor. We can also estimate at this point the cumulative error after the launch is completed. We will be throwing balls on this rotor to increase its velocity until it becomes comparable to the velocity of the incoming balls, $\vec{v}_{in}$. According to Equation (8) again, the total number of these increments would be $M/2m$. So, the total error can now be determined by multiplying the total number of increments by the stepwise error. The cumulative error is estimated to be of the order of $m/2M$.

In the case of an infinite rotor mass, the rotational energy gained by the relay is zero, making the cases of relay and payload similar. Furthermore, if we start for this first ball launch with an initial zero velocity for the relay ($V_{in} = 0$), we get from Equation (7):

$$\vec{v}_{out} \sim -\vec{v}_{in},$$

and from Equation (8):

$$\vec{V}_{out} \sim 2\frac{m}{M}\vec{v}_{in},$$

which represents the limiting case of an elastic collision where the ball transfers twice its momentum to the relay. Now, we can determine the actual rotational energy if we consider a step by step analysis and write the angular momentum conservation equation. The angular momentum lost by the ball will be gained by the rotor. Knowing $\vec{v}_{in}$ and $\vec{v}_{out}$ from the Lagrangian analysis, we can write the angular momentum $\vec{L}$ before and after the interaction

respectively as:

$$\vec{L}_{in} = \vec{r}_{in} \times m\vec{v}_{in},$$

and

$$\vec{L}_{out} = \vec{r}_{out} \times m\vec{v}_{out},$$

where $\vec{r}_{in}$ is the position vector when the ball impacts the track on entry, and $\vec{r}_{out}$ its position at the exit of the track when traveling back to the primary launcher. Then, the decrease in the ball angular momentum, $\vec{L}_{in} - \vec{L}_{out}$ will correspond to the increase in angular momentum of the relay, $\Delta \vec{L}_{rotor}$. And finally, if we know the moment of inertia of the rotor, $\mathcal{I}_{rotor}$ we will determine the increase in its angular velocity $\Delta\omega$ from:

$$\Delta\vec{L}_{rotor} = \mathcal{I}_{rotor}\Delta\vec{\omega}.$$

2.2.2 Momentum Transfer after the Launch of n Balls

Let us now consider a sequence of such interactions and derive a recursive formula giving the increase in relay/payload velocity as a function of the number of balls launched. The relay/payload initial velocity, before the launch of the first ball, is V_{in}. After the impact of the first ball, the velocity becomes:

$$\vec{V}_{in} + 2\frac{m}{M}\frac{(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}},$$

as shown in Equation (8). We can now write the final velocity of the relay/payload after the second ball is launched:

$$\vec{V}_{in} + 2\frac{m}{M}\frac{(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}} + 2\frac{m}{M}\frac{(\vec{v}_{in} - \vec{V}_{in} + 2\frac{m}{M}\frac{(\vec{v}_{in}-\vec{V}_{in})}{1+\frac{m}{M}})}{1 + \frac{m}{M}},$$

or,

$$\vec{V}_{in} + 2\left[2\frac{m}{M}\frac{(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}}\right] - \left[2\frac{m}{M}\right]^2\frac{(\vec{v}_{in} - \vec{V}_{in})}{\left[1 + \frac{m}{M}\right]^2}.$$

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We also derived the result for the case of three balls launched ($n = 3$):

$$\vec{V}_{in} + 3\left[2\frac{m}{M}\frac{(\vec{v}_{in} - \vec{V}_{in})}{1 + \frac{m}{M}}\right] - 3\left[2\frac{m}{M}\right]^2\frac{(\vec{v}_{in} - \vec{V}_{in})}{\left[1 + \frac{m}{M}\right]^2} + \left[2\frac{m}{M}\right]^3\frac{(\vec{v}_{in} - \vec{V}_{in})}{\left[1 + \frac{m}{M}\right]^3}.$$

We went on and wrote the velocity resulting after 4 balls have been launched, and also after 5 balls were launched. We then expressed the general formula giving the final velocity of the relay/payload after n balls are launched as:

$$\vec{V}_{in} + \sum_{p=1}^{n}\binom{n}{p}(-1)^{p+1}\left[2\frac{m}{M}\right]^p\frac{(\vec{v}_{in} - \vec{V}_{in})}{\left[1 + \frac{m}{M}\right]^p}. \tag{9}$$

We then wrote a little program (cf. APPENDIX), that uses the expression derived above and computes the velocity of the relay after n balls are launched. We plotted in Figure - 5 the generated data to show how $\vec{V}_{out}$ varies with n. We can see that the velocity of the relay increases almost linearly with the number of balls launched, but then, as the relay is going faster and faster, the effect of one ball becomes less and less important. Very little momentum is transferred when the rotor is moving at speeds similar to the speed of the incoming balls, until it reaches the speed of the balls and they will not be able to catch up with it. In the practical case considered here, some 8000 balls would be needed to reach a velocity $v_{relay} = 1698$ m/s. Only the first 10% of the total number of balls needed to reach this limiting case of a relay moving, practically as fast as the balls, will provide half of the final rotor speed while the remaining 90% of the balls will provide the remaing half of the total velocity increments. This suggests that one should accelerate the relay until it reaches about half the velocity of the incoming
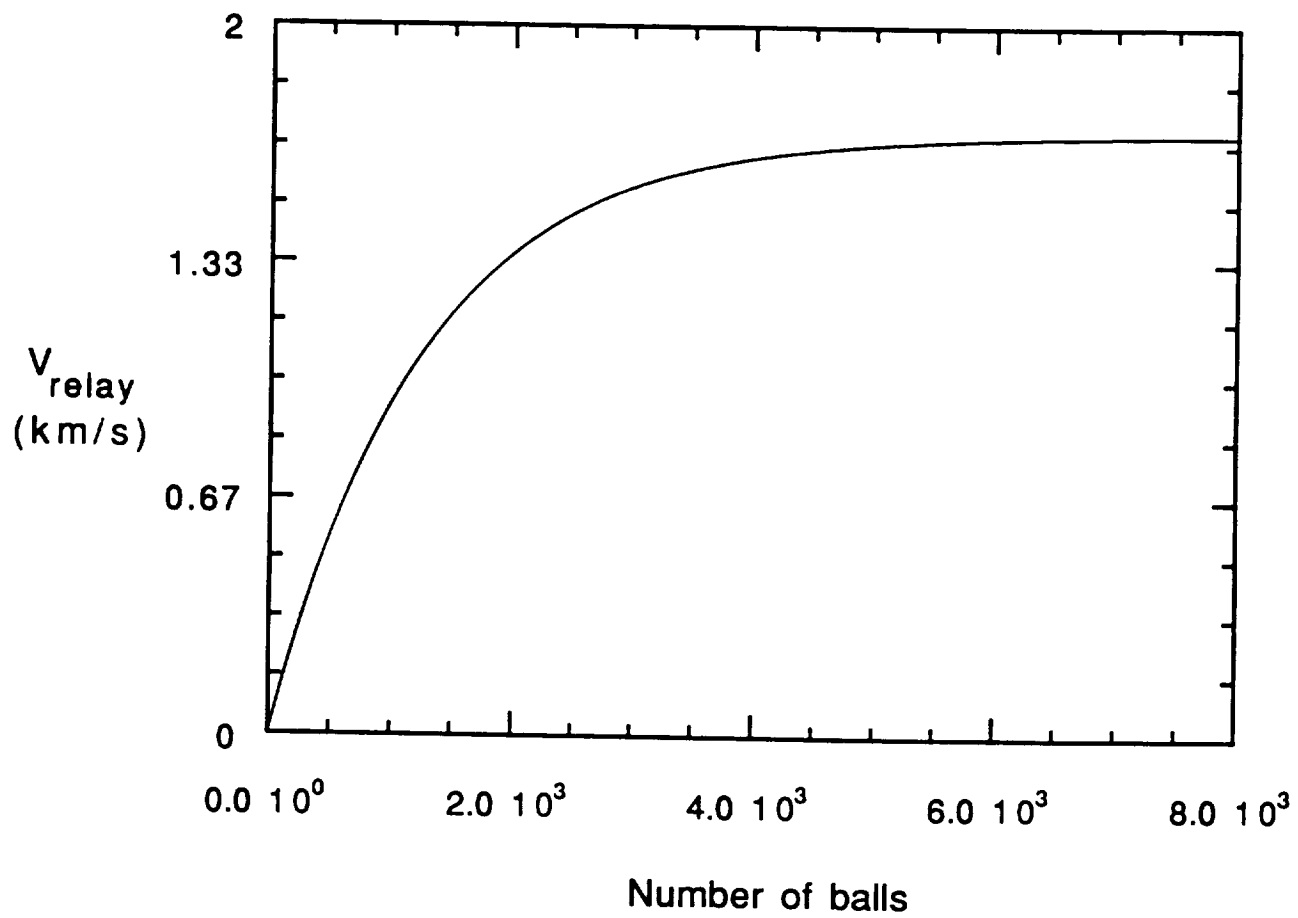
Figure - 5. Velocity of the relay versus the number of balls launched.

balls as further acceleration would not be cost effective since the balls are not as efficient in terms of momentum transfer.

2.3 DESIGN CONSIDERATIONS FOR THE CENTRIFUGAL RELAYS

One major design consideration is the fact that the relay must process a large number of balls quickly and efficiently. The rotor is a disk or a cylinder spinning at high angular velocity with guide tracks to process the balls in the necessary direction. The balls are either caught near the axis of the rotor and launched forward or caught nearer the tip of the rotor, slowed to a stop at the axis and then propelled forward or backward using another track. The objective of the first type of processing is to increase the velocity of the balls. The second type of processing increases the angular and linear momentum of the relay. For the determination of the exit velocity and the forces of constraint on the ball, a track shape needs to be chosen. If the ball starts at the center of the rotor and is launched tangentially, it leaves with twice the tip speed of the rotor, where the maximum tip speed depends on material properties of the rotor [23]. The shape analyzed here is the logarithmic spiral. The objective is to develop a method that can be used to analyze different shapes quickly and easily.

Original studies of the kinematics of ball/rotor interaction idealized the relatively small balls as point masses [24]. Then an equation of motion treating the pellet as a ball of finite radius was derived [29]. One constraint to be satisfied is that the balls must not bounce nor slide during the catching process to insure a maximum exchange of momentum between the ball and the rotor. This constraint is satisfied if the velocity vectors of the point of impact on the ball and the blade are equal. We have started detailed kine-

matic studies by assuming that the velocity vector at the edge of a ball rolled off a previous rotor must match the velocity vector of the contact point on the next rotor it encounters. This condition is met by slightly adjusting the aiming and timing so each ball launched encounters the next rotor at the correct distance from this rotor's axis. This removes a degree of freedom which complicated the analysis made in the paper "Centrifugal Relays for Spacecraft Propulsion" [24]. This should considerably simplify the next step in the kinematic analysis–writing a computer program which follows the trajectories of all objects in the system in the presence of a gravitational field.

To obtain an even more complete description of ball/rotor interactions, AUTOLEV program may be used. This program, by Schaechter and Levinson [30], automatically generates FORTRAN coding for complete solution for rigid body kinematics of finite mass balls interacting with a free-flying rotor in a gravitational field. While including the effects of finite ball/rotor mass ratio is not expected to be important for the kinematics of the overall mission performance, it will determine the exact amount of magnetic drag needed to hold the balls near the rotor axis long enough that they will roll back off at the right time. The same method can also be used to examine various schemes for emplacing and activating these coils. To date, the AUTOLEV program has been successfully used for balls landing with properly matched velocities, rolling up towards the axis of a rotating relay, and rolling back off the track for relaunch [31]. The program is designed so that investigation of additional effects like small velocity mismatches at catch, finite ball/rotor mass ratios, and gravitational fields are straightforwardly accomodated by changes in the program input.

2.4 CONCEPTUAL DESIGN OF THE RELAYS

The first task in designing the centrifugal relays is coming up with a conceptual idea. The rotor has to have guide tracks for the pellets. It should spin at very high angular velocity processing balls in both directions. The rotor should include guidance tubes in the path of the incoming pellets for position control. A protection shield has to be placed in front of the guide tubes and rotor to prevent errant balls from impacting these systems. A supporting truss structure is needed to mate the rotor, the guidance tubes and the protection shield. To minimize the total system mass, the size and the mass of the whole relay may need to be kept small. This conceptual design can be summarized schematically as illustrated in Figure - 6 [32].

2.5 DESIGN CONSIDERATIONS FOR THE RO-TOR

The basic concept of a centrifugal projectile accelerator is that a projectile can be accelerated centrifugally by either being constained to move along a channel in a rotating disk or arbor, or by being released from the edge [23]. Figure - 7 illustrates such launcher [23].

The rotor is a disk or a cylinder spinning at high angular velocity with guide tracks to process the balls in the necessary direction. The "forward" or "backward" launching depends on the objective of the launch. It could be increasing the velocity of the ball, or the angular and linear momentum of the relay. The idea in this design is to have one rotor accomplish one task. For example one rotor would catch the balls near the tip and slow them to a stop at its axis increasing the rotational and transitional velocity of the relay. The balls would be collected at the axis of this rotor and then fed to
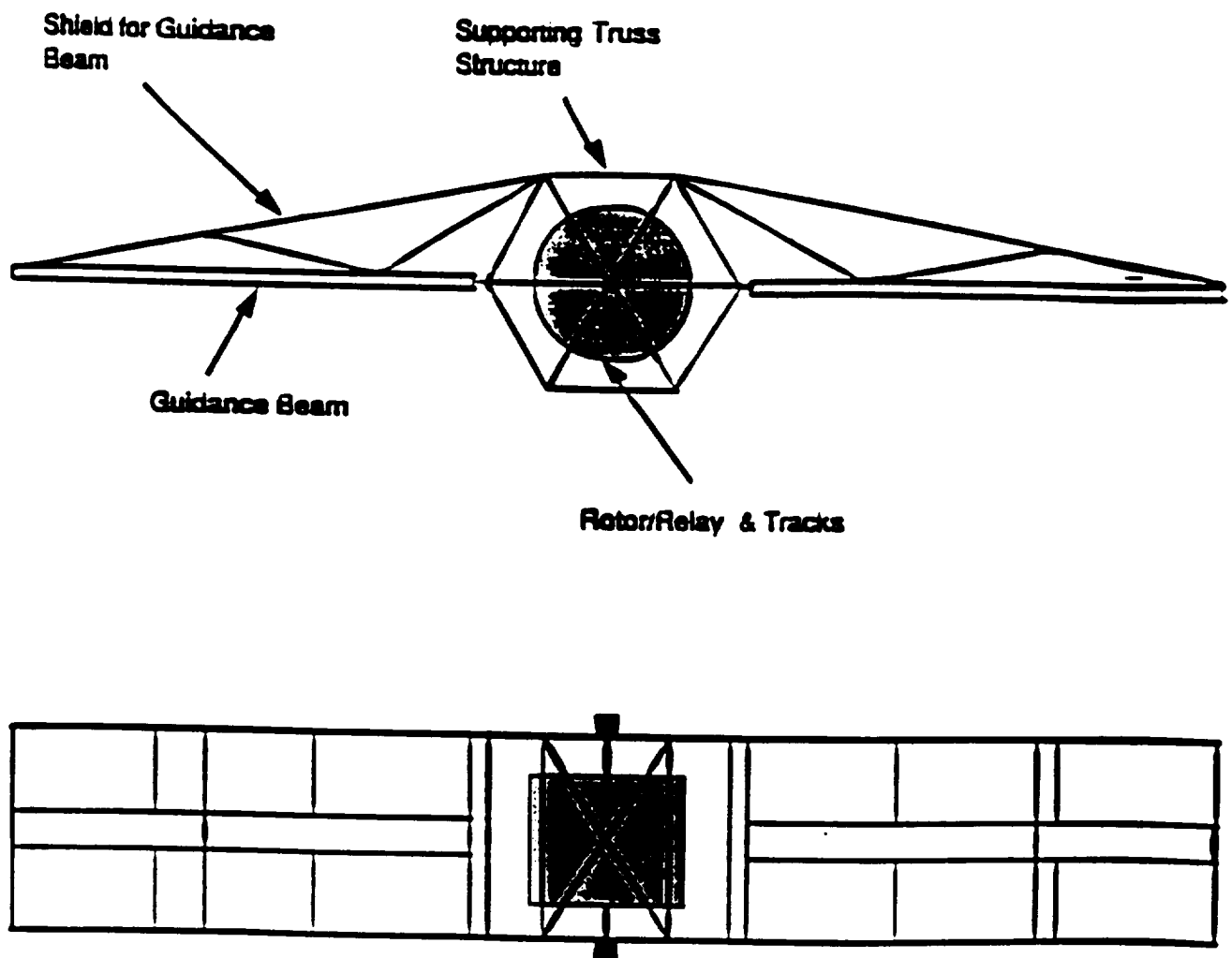
24

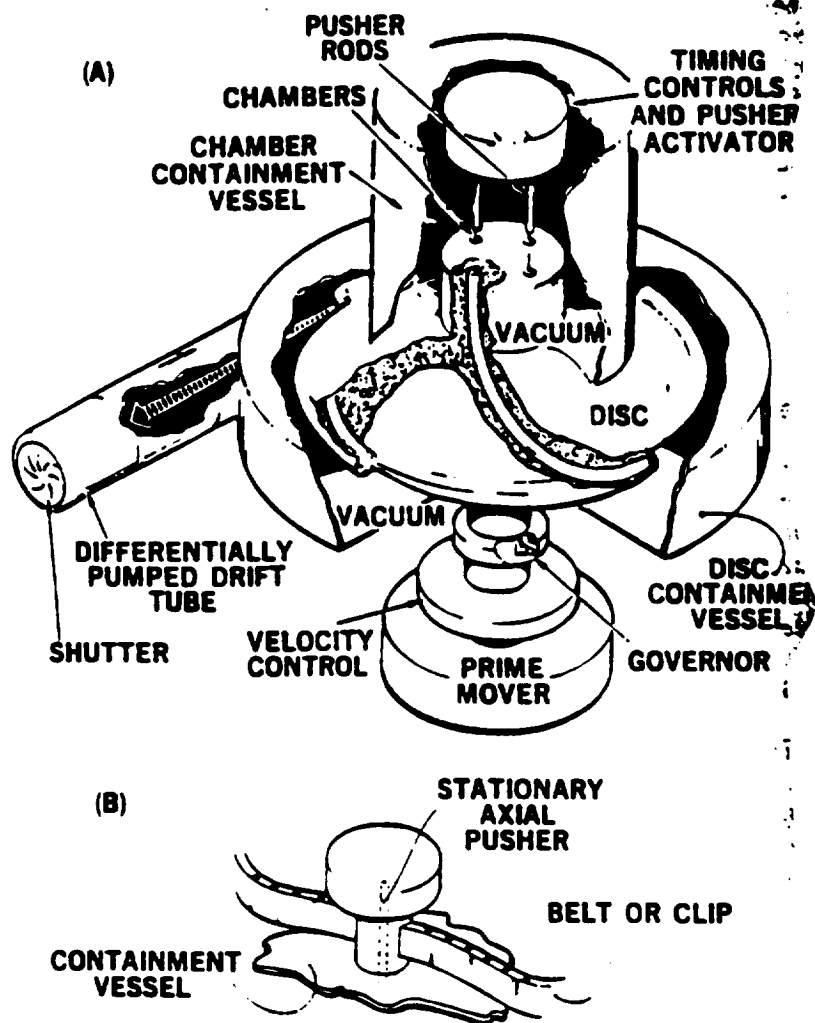Figure - 6. Elevation and plan view of a centrifugal relay.

Figure - 7. Channeled disc centrifugal projectile activator.
(a) System with corotating chamber feed.
(b) On axis belt or clip feed.

another disk to be propelled in the necessary direction (forward or backward). A display of this concept is given in Figure - 8 [32]. The disks are all rotating at the same rate. The balls are fed through the shafts to the correct track. In practice, instead of having several disks, we will just use a multiple track rotor.

Another point discussed here is whether the rotor should be a cylinder with a track bored through it or a rotating disk with a surface track. The latter case may seem more interesting for reasons of machining and catching incoming balls. It is easier to catch the ball on a surface track then in a small hole. But the track bored through a cylinder may be more practical for the transfer of the ball from one track to the other at the center. Conceptually, the best design would be a combination of these two alternatives. We would have a surface track starting at the edge of the rotor and continuing into a path bored through the rotor. The track will end at the axis of the cylinder. The second track will start from the center at a lower level and emmerge to the surface toward the tip of the rotor.

Typical materials of interest for use in a pellet launcher are given in Table - 1 [33], where V_c represents the critical velocity, ρ is the density and σ the yield strength of the rotor material. To build a launcher that ejects pellets with velocities less than V_c, we can just use a tube of uniform cross section. Above V_c, the pellet launcher must be tapered for part of its length, increasing in thickness toward the axis [33].

The tube may be subject to considerable wear due to friction and abrasion from the pellets, and should be designed for easy maintenance. This could be accomplished by providing the tube with a removable liner, and by designing other high - wear parts for easy removal and replacement [33].

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Ball can be
dropped here
and thrown forward

ball can be dropped here
and thrown back

Ball caught at tip of
one rotor
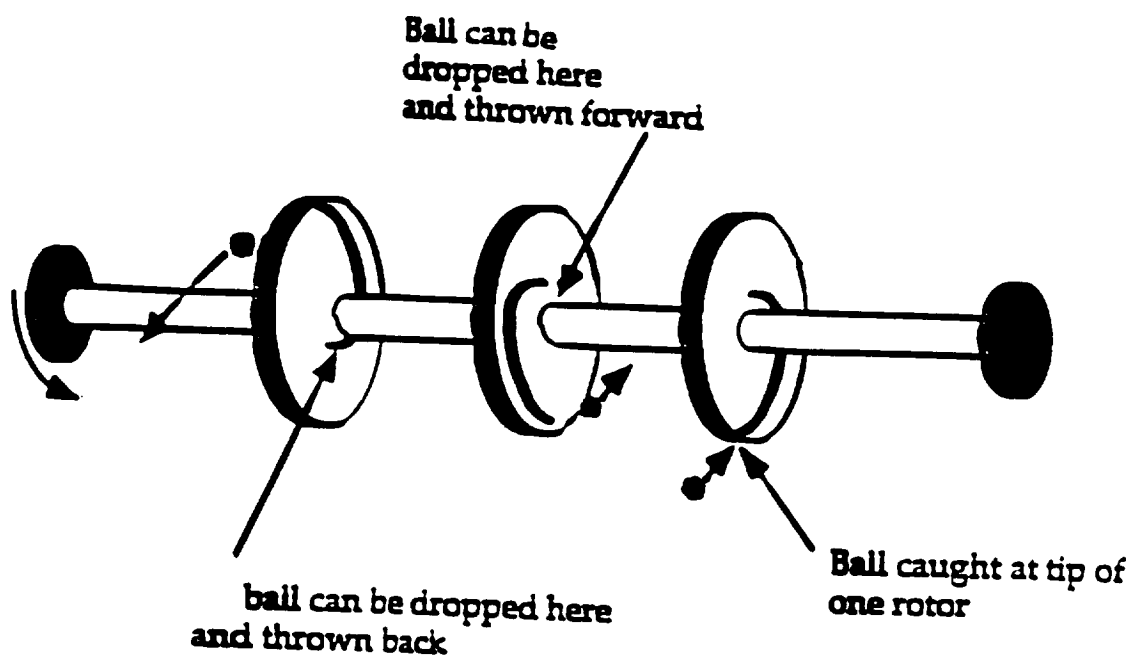
Figure - 8. Multiple track relay concept.

Material	e,g/cm^3	Sy,10^3MPA	V_c,m/s
Maragin steel	8.0	2.76	831
E glass	2.5	3.45	1631
Carbon fiber	1.4	2.76	1987
S glass	2.5	5.17	2036
Kevlar-49	1.45	3.62	2237
Fused silica	2.2	13.8	3545

Table - 1. Materials of interest for
a rotary pellet launcher

National Laboratory the lead laboratory in its flywheel rotor and technology development program. Although the objective of that program was storing energy, the rotors that were developed can be modified to launch projectiles [23]. One of the near-term objectives of this program is to develop an efficient, economical, and practical composite flywheel. State-of-the-art composite rotors developed in the program [34] have maximum tip speeds $\leq$ 1 km/s. Their radii are typically 10 − 20 cm, and masses are a few kilograms. These centrifugal launchers are summarized in Table - 2 [23].

For the determination of the exit velocity and the forces of constraint on the ball, a track shape needs to be chosen. The launch should be such that the exit velocity is approximately twice the tip speed of the rotor. The maximum tip speed, V_c is proportional to $(\sigma/\rho)^{1/2}$ [23].

The shape considered is the logarithmic spiral. An illustration of the track shape is given in Figure - 9. The method developed should be general and could be applied to analyze different shapes quickly and easily. This is the subject of the next chapter, a detailed analysis of the launching process.

Rotor	State-of-the-Art	Near-Term-DOE Objective	Demo
Maximum energy density (MJ/kg)	0.24	0.32	0.57
Maximum rotational energy (MJ)	0.62	3.6	3.4
Mass (kg)	2.6	11.2	6
Radius (cm)	20.3	30.5	120
Maximum frequency (rpm)	49,000	35,400	12,000
Maximum tip speed (km/s)	1.04	1.13	1.5

Projectile	State-of-the-Art	Near-Term-DOE Objective	Demo
Mass (g)	26	112	60
Maximum velocity (km/s)	1.04	1.13	3
Maximum energy (MJ)	0.014	0.072	0.27

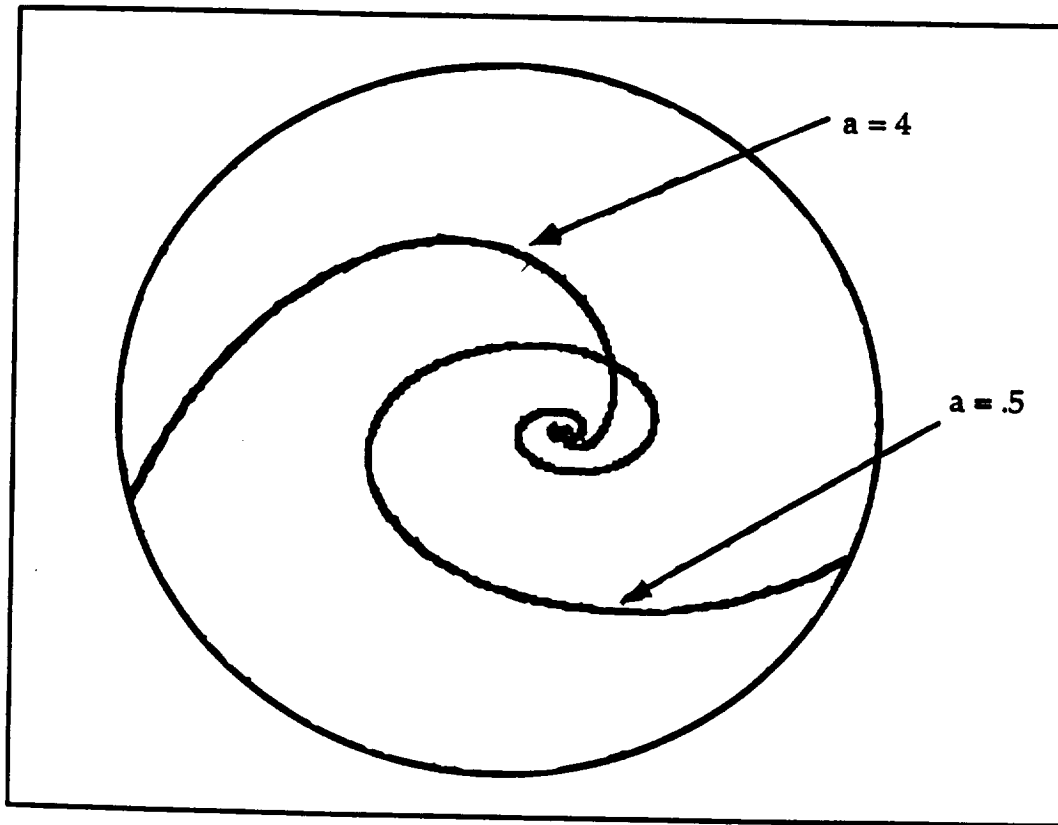Table - 2. Rotors and launching capabilities

Figure - 9. Logarithmic spiral tracks.

Chapter 3

ANALYSIS OF THE LAUNCHING PROCESS

Lagrange equations with multipliers are used to analyze the launch process. The track shape is represented as a constraint equation between the radial and angular positions. A second constraint equation represents the fact that the ball is restricted to roll on the track. We wish to avoid skidding because frictional losses would make the system non-conservative.

The velocity of the center of mass of the ball along the track is:

$$v^2 = \dot{r}^2 + r^2\dot{\theta}^2, \tag{10}$$

and

$$\theta(t) = \theta_0 + \omega t + \zeta(t), \tag{11}$$

where r is the radial displacement of the ball, ω the rotation speed of the rotor, ζ the angle that defines the position of the ball along the track, and θ_0 any initial angle that might be present as illustrated in Figure - 10. If we let m be the mass of the ball, then the kinetic energy is:

$$T = \frac{1}{2}m\left[\dot{r}^2 + r^2(\omega + \dot{\zeta})^2\right] + \frac{1}{2}\mathcal{I}\dot{\alpha}^2, \tag{12}$$

33

Track: $r = r_0 e^{a\zeta}$
$a = 1$
$0 < \zeta < 2.3$ rad
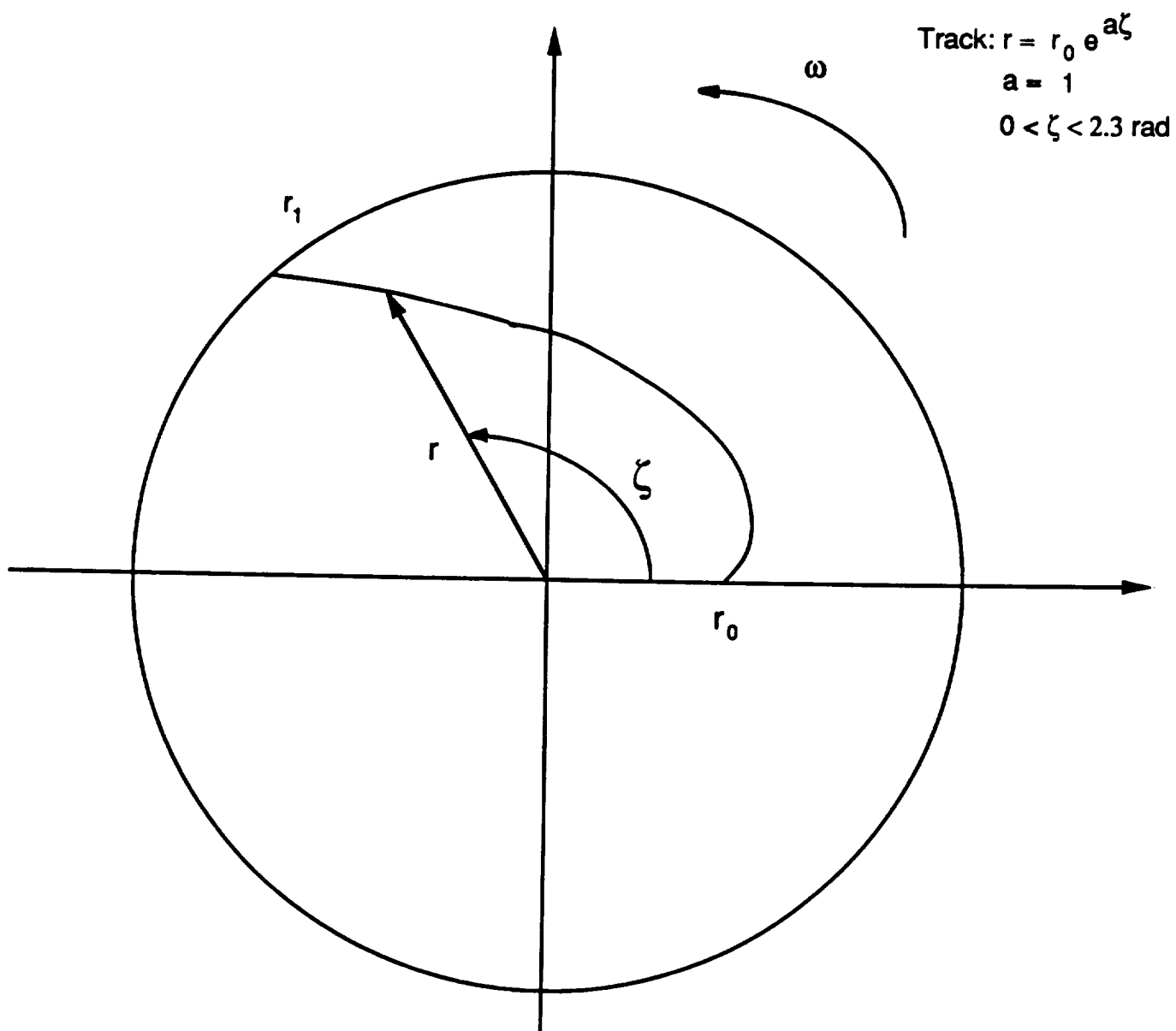
Figure - 10. Track on primary launcher.

where $\mathcal{I}$ is the moment of inertia of the ball defined as

$$\mathcal{I} = \frac{2}{5}m\ell^2, \tag{13}$$

ℓ being the radius of the ball and $\dot{\alpha}$ the angular velocity of the ball around its center of mass. These quantities are all defined in Figure - 11.

The Lagrangian is:

$$L = T - V = T \tag{14}$$

since there is no potential energy.

The constraints equations are:

$$f_1 = r_0 e^{a\zeta(t)} - r(t) = 0, \tag{15}$$

and

$$f_2 = \ell\alpha(t) - s(t) = 0, \tag{16}$$

where s, the track shape length defined in Figure - 11, is related to r as follows:

$$ds^2 = dr^2 + r^2 d\zeta^2, \tag{17}$$

or,

$$\left(\frac{ds}{dr}\right)^2 = 1 + r^2 \left(\frac{d\zeta}{dr}\right)^2. \tag{18}$$

Now, from Equation (15), we can write:

$$\zeta = \frac{1}{a}\ln\frac{r}{r_0} \tag{19}$$

$$\implies \frac{d\zeta}{dr} = \frac{1}{ar}. \tag{20}$$

Then,

$$\left(\frac{ds}{dr}\right)^2 = 1 + \frac{1}{a^2}, \tag{21}$$

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S (track distance)

α (t)

r(t)

ζ (t)

Center of relay

I (radius)

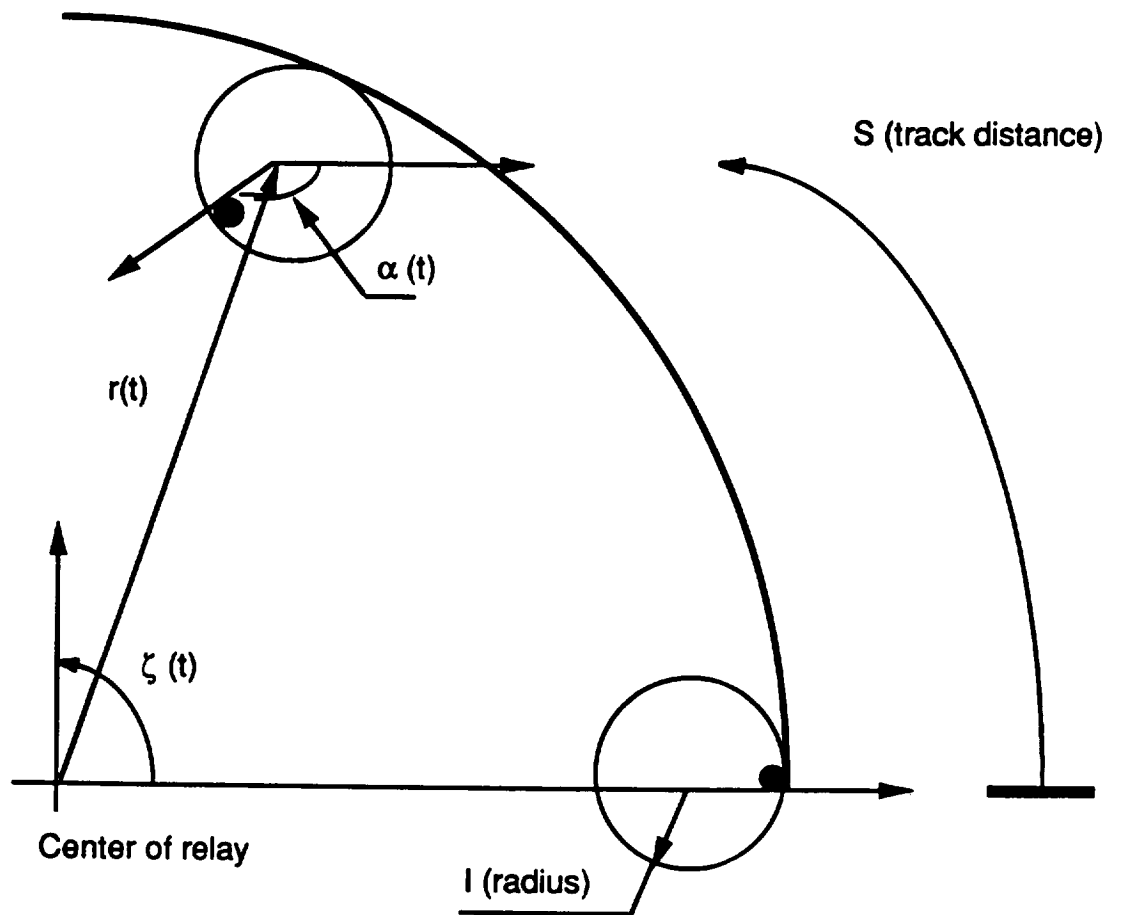Figure - 11. Rolling of the ball on the track.

or,

$$ds = \left(\sqrt{1 + \frac{1}{a^2}} \right) dr. \tag{22}$$

Integrating this equation and using the initial condition $s = 0$ when $r = r_0$ to determine the constant of integration, we can write $s(t)$ as a function of $r(t)$:

$$s = \left(\sqrt{1 + \frac{1}{a^2}} \right)(r - r_0), \tag{23}$$

and f_2 becomes:

$$f_2 = \ell \alpha(t) - \left(\sqrt{1 + \frac{1}{a^2}} \right)(r(t) - r_0) = 0. \tag{24}$$

The equations of motion are:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{r}} \right) - \frac{\partial L}{\partial r} = \lambda_1 \frac{\partial f_1}{\partial r} + \lambda_2 \frac{\partial f_2}{\partial r}, \tag{25}$$

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\zeta}} \right) - \frac{\partial L}{\partial \zeta} = \lambda_1 \frac{\partial f_1}{\partial \zeta} + \lambda_2 \frac{\partial f_2}{\partial \zeta}, \tag{26}$$

and

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\alpha}} \right) - \frac{\partial L}{\partial \alpha} = \lambda_1 \frac{\partial f_1}{\partial \alpha} + \lambda_2 \frac{\partial f_2}{\partial \alpha}. \tag{27}$$

Dividing Equation (25) by m gives:

$$\ddot{r} - r(\omega + \dot{\zeta})^2 + \frac{\lambda_1}{m} + \frac{\lambda_2}{m} \sqrt{1 + \frac{1}{a^2}} = 0. \tag{28}$$

Equation (26) can be written as:

$$r^2 \ddot{\zeta} + 2r\dot{r}(\omega + \dot{\zeta}) - ar\frac{\lambda_1}{m} = 0. \tag{29}$$

Dividing Equation (27) by $\mathcal{I}$ gives:

$$\ddot{\alpha} - \frac{\lambda_2}{\mathcal{I}}\ell = 0. \tag{30}$$

From Equation (30) combined with Equations (16, 23) differentiated twice with respect to time, we get:

$$\lambda_2 = \frac{\mathcal{I}}{\ell}\ddot{\alpha} = \frac{\mathcal{I}}{\ell^2}\ddot{s} = \left(\sqrt{1 + \frac{1}{a^2}}\right)\frac{\mathcal{I}}{\ell^2}\ddot{r}. \tag{31}$$

Solving for λ_1 from Equation (29) we get:

$$\frac{\lambda_1}{m} = \frac{2\omega}{a}\dot{r} + \frac{2}{a}\dot{r}\dot{\zeta} + \frac{r}{a}\ddot{\zeta}. \tag{32}$$

Now, substituting for λ_1 from above and λ_2 from Equation (31), and using the following set of equations derived from Equation (10)

$$\dot{\zeta} = \frac{\dot{r}}{ar}, \tag{33}$$

and

$$\ddot{\zeta} = \frac{1}{a}\frac{r\ddot{r} - \dot{r}^2}{r^2}, \tag{34}$$

Equation (28) becomes:

$$\ddot{r}\left[1 + \frac{\mathcal{I}}{m\ell^2}\left(1 + \frac{1}{a^2}\right) + \frac{1}{a^2}\right] +$$

$$\dot{r}^2\left[-\frac{1}{a^2r} + \frac{2}{a^2r} - \frac{1}{a^2r}\right] + \dot{r}\left[-\frac{2\omega}{a} + \frac{2\omega}{a}\right] - r\omega^2 = 0, \tag{35}$$

or simply,

$$\ddot{r} - K^2r = 0, \tag{36}$$

where

$$K^2 = \frac{\omega^2}{\left(1 + \frac{1}{a^2}\right)(1 + \mathcal{J})}, \tag{37}$$

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and

$$J = \frac{\mathcal{I}}{m\ell^2}.$$

We can also write K^2 as:

$$K^2 = \frac{5\omega^2}{7\left(1 + \frac{1}{a^2}\right)}, \tag{38}$$

taking into account the fact that the projectiles have a spherical shape, which means:

$$J = \frac{2}{5}.$$

The corresponding solution to the governing equation is:

$$r(t) = A \cosh K(t - t_0) + B \sinh K(t - t_0). \tag{39}$$

Differentiating this equation, we can write:

$$\dot{r}(t) = AK \sinh K(t - t_0) + BK \cosh K(t - t_0). \tag{40}$$

Now, at $t = t_0$, when the ball is released near the center, we have:

$$r(t_0) = r_0,$$

and

$$\dot{r}(t_0) = 0.$$

Consequently,

$$A = r_0,$$

and

$$B = 0.$$

The trajectory of the ball on the track is then given by:

$$r(t) = r_0 \cosh K(t - t_0). \tag{41}$$

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One quantity of interest is the exit velocity. It can now be determined, and we can study its behavior as a function of the design parameters such as the track shape constant a.

3.1 EXIT VELOCITY

The position of the ball on the rotating track with respect to a fixed frame, with $\theta_0 = 0$, is given at all times by:

$$\vec{r}(t) = \begin{pmatrix} r_0 e^{a\zeta(t)} \cos(\omega t + \zeta(t)) \\ r_0 e^{a\zeta(t)} \sin(\omega t + \zeta(t)) \end{pmatrix}. \tag{42}$$

We can get the velocity vector by differentiating the position vector above,

$$\vec{v}(t) = \begin{pmatrix} r_0 a\dot{\zeta} e^{a\zeta(t)} \cos(\omega t + \zeta(t)) - r_0(\omega + \dot{\zeta})e^{a\zeta(t)} \sin(\omega t + \zeta(t)) \\ r_0 a\dot{\zeta} e^{a\zeta(t)} \sin(\omega t + \zeta(t)) + r_0(\omega + \dot{\zeta})e^{a\zeta(t)} \cos(\omega t + \zeta(t)) \end{pmatrix}. \tag{43}$$

Then,

$$v^2 = r_0^2 e^{2a\zeta} \left[(\dot{\zeta} + \omega)^2 + a^2 \dot{\zeta}^2 \right]. \tag{44}$$

Using Equation (33), v^2 can be written as:

$$v^2 = r^2 \left[\left(\frac{\dot{r}}{ar} + \omega \right)^2 + a^2 \left(\frac{\dot{r}}{ar} \right)^2 \right]. \tag{45}$$

Now, from Equation (41) we get:

$$\dot{r} = Kr_0 \sinh K(t - t_0). \tag{46}$$

This can be combined with Equation (41) to express a relationship between r and $\dot{r}$,

$$\dot{r} = K\sqrt{r^2 - r_0^2}. \tag{47}$$

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Substituting for $\dot{r}$ from Equation (47) and K from Equation (38) in Equation (45) we get:

$$v^2 = r^2\omega^2 \left[1 + \frac{5}{7}\left(1 - \left(\frac{r_0}{r}\right)^2\right) + 2\sqrt{\frac{5}{7}}\sqrt{\frac{1 - \left(\frac{r_0}{r}\right)^2}{1 + a^2}} \right].$$ (48)

And finally, we can write the exit velocity v_{exit} as:

$$v_{exit} = v_{tip} \left[1 + \frac{5}{7}\left(1 - \left(\frac{r_0}{r_1}\right)^2\right) + 2\sqrt{\frac{5}{7}}\sqrt{1 - \left(\frac{r_0}{r_1}\right)^2} \frac{1}{\sqrt{1 + a^2}} \right]^{1/2},$$ (49)

where v_{tip} is equal to ωr_1.

Also from the geometry, as outlined on Figure - 12, we can write:

$$\tan\gamma = \frac{dy}{dx} = \frac{dy/d\zeta}{dx/d\zeta},$$

where $\tan\gamma$ is the slope of the tangent to the track at the exit, and x and y are given by:

$$x = e^{a\zeta}\cos(\zeta + \omega t),$$

and

$$y = e^{a\zeta}\sin(\zeta + \omega t).$$

We can also write $\tan\gamma$ as:

$$\tan\gamma = \frac{a\sin(\zeta + \omega t) + \cos(\zeta + \omega t)}{a\cos(\zeta + \omega t) - \sin(\zeta + \omega t)}.$$

Now, let us express ζ in terms of γ. We can write, after inspecting Figure - 12:

$$\zeta + \omega t = \beta + \gamma - \frac{\pi}{2},$$

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$$\frac{\pi}{2} = \beta + [\pi - \zeta - (\pi - \gamma)]$$

$$= \beta + \gamma - \zeta$$

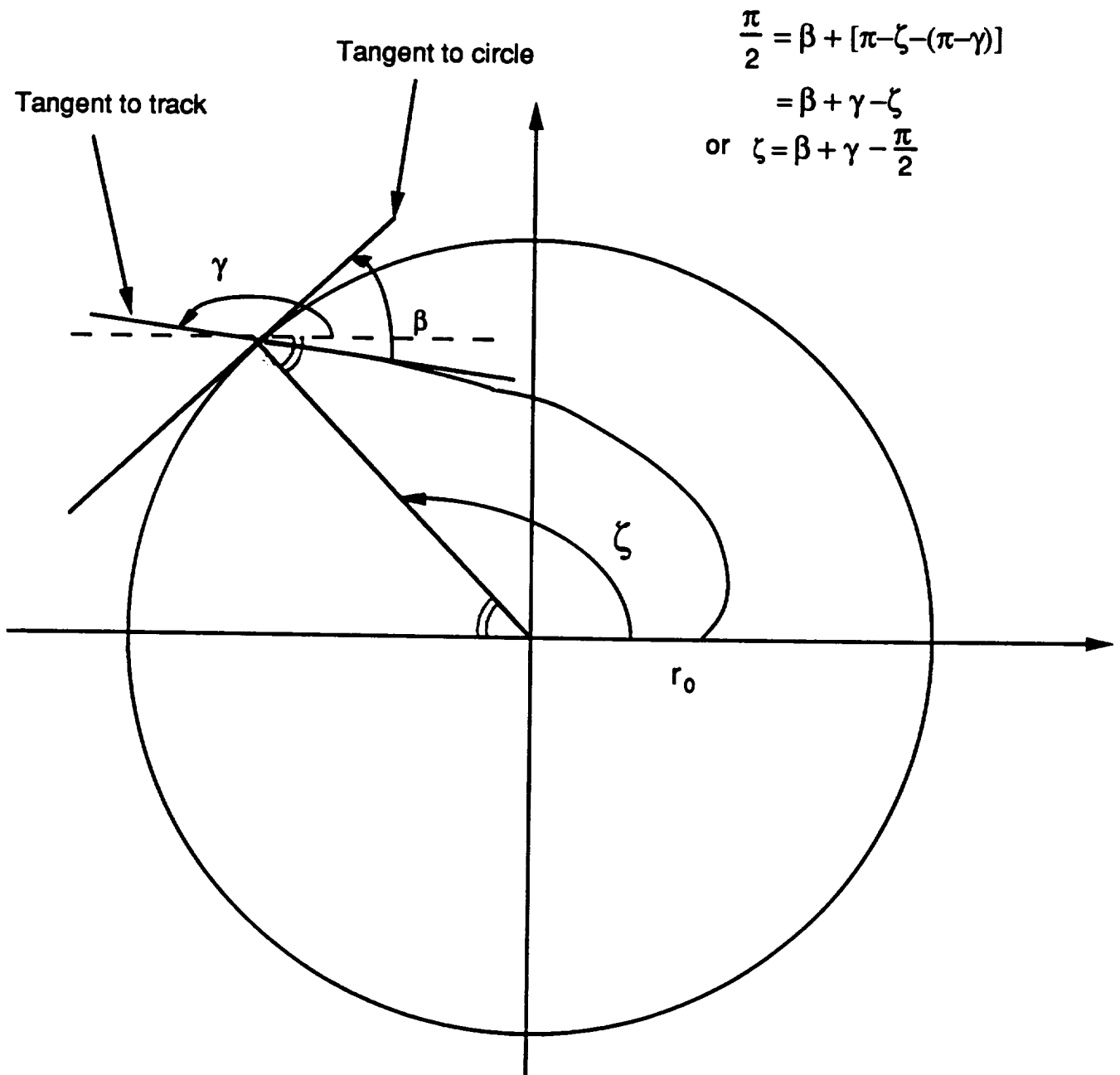$$\text{or} \quad \zeta = \beta + \gamma - \frac{\pi}{2}$$

Figure - 12. Angular information of the ball at the exit from the primary launcher.

where β is the angle between the tangent to the rotor rim and the tangent to the track at the exit. Using this relationship between the angles, $\tan \gamma$ becomes:

$$\tan \gamma = \frac{-a\cos(\beta + \gamma) + \sin(\beta + \gamma)}{a\sin(\beta + \gamma) + \cos(\beta + \gamma)} = \frac{\sin \gamma}{\cos \gamma}.$$

This equation can be rewritten as:

$$-a\cos(\beta + \gamma)\cos\gamma + \sin(\beta + \gamma)\cos\gamma = a\sin(\beta + \gamma)\sin\gamma + \cos(\beta + \gamma)\sin\gamma,$$

or simply,

$$-a\cos\beta = -\sin\beta.$$

Thus, the main result of this geometrical derivation is:

$$\tan\beta = a$$

or,

$$\cos\beta = \frac{1}{\sqrt{1 + a^2}}.$$

If the ball leaves from the center ($r_0 = 0$) tangentially ($\beta = 0$) Equation (49) becomes:

$$v_{exit} = 1.84 v_{tip},$$

and we can see that the ball leaves with almost twice the tip speed of the launcher. Felber [23] stated that the ball leaves with twice the tip speed in the same conditions of launch. But in his case, the ball was considered as a point mass with zero inertia ($\ell \to 0$), which means that all the energy gained by the ball is used for the translational displacement of the ball, whereas in our case, some of the energy is used to spin the ball around its center of mass. So, the resulting translational velocity is less than in the case of

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Felber's calculations, and in general, the exit velocity does depend on the inertia of the projectile. We can actually find his result from our general expression just by setting $\mathcal{J} = 0$ (no inertia) in Equation (37) which gives:

$$K^2 = \frac{\omega^2}{1 + \frac{1}{a^2}}.$$

Then, substituting for $\dot{r}$ from Equation (47) and K from above in Equation (45), we find:

$$v_{exit} = v_{tip} \left[2 - \left(\frac{r_0}{r_1}\right)^2 + 2\sqrt{1 - \left(\frac{r_0}{r_1}\right)^2} \frac{1}{\sqrt{1 + a^2}} \right]^{1/2},$$

or, as a function of the angle β at launch,

$$v_{exit} = v_{tip} \left[2 - \left(\frac{r_0}{r_1}\right)^2 + 2\sqrt{1 - \left(\frac{r_0}{r_1}\right)^2} \cos\beta \right]^{1/2},$$

which is what Felber [23] published. In this particular case, we can see that when the ball leaves from the center ($r_0 \to 0$) tangentially ($\beta = 0$), it leaves with twice the tip speed ($v_{exit} = 2v_{tip}$).

The dependency of the exit velocity on the track shape constant a is illustrated in Figure - 13.

Another important point about this launch process analysis is the determination of the time delay needed to insure that the ball is launched exacly forward. The ball will be kept near the rotor axis while launcher is rotating until the track is oriented in such a way that when the ball rolls off the edge it is going exactly forward.
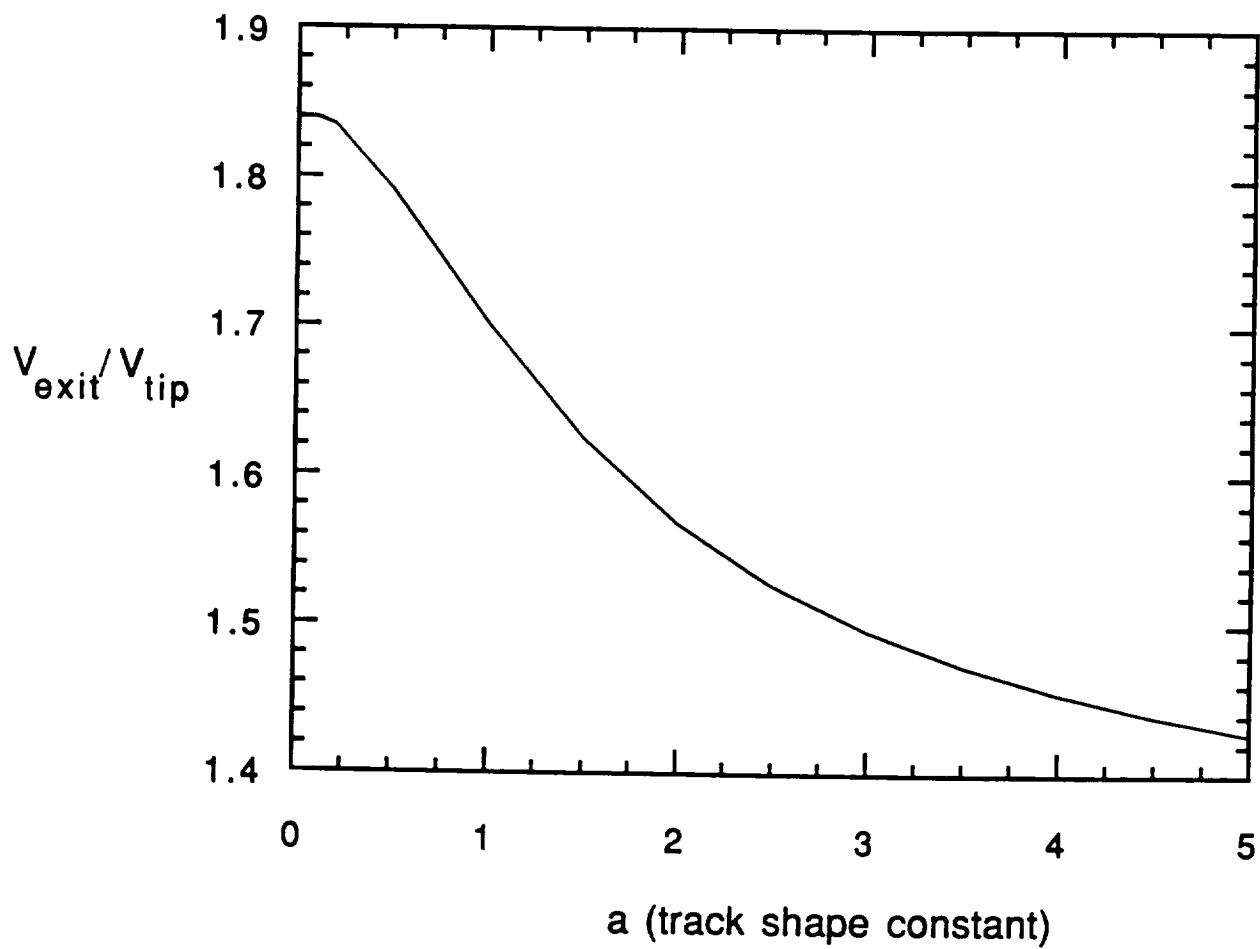
Figure - 13. Exit velocity of the ball versus the track
shape constant.

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3.2 TIME DELAY NEEDED TO INSURE LAUNCH EXACTLY FORWARD

The position of the ball on the rotating track is given by Equation (42) and its velocity by Equation (43). In order for the ball to go exactly forward (horizontally), $\vec{v}$ at the exit should be of the form:

$$\vec{v}_{exit} = \begin{pmatrix} v_{exit} \\ 0 \end{pmatrix}. \tag{50}$$

Let t_1 be the time at which the ball leaves the launcher, and t_0 the time at which the ball is released near the center of the rotor. The origin of time ($t = 0$), is taken to be at the beginning of the spinning process of the launcher. We can determine the time it takes for the ball to roll down the track all the way to the edge, which is ($t_1 - t_0$), by solving Equation (41) at the exit ($t = t_1$), where $r(t)$ will be set equal to r_1. Then we can get the total launch time t_1 by setting Equation (43) at the exit equal to Equation (50). Finally, the delay time t_0 can be determined just by combining these two results.

3.2.1 Determination of $(t_1 - t_0)$

At the exit, Equation (41) becomes:

$$r(t_1) = r_1 = r_0 \cosh K(t_1 - t_0). \tag{51}$$

This equation can be solved for $(t_1 - t_0)$ as:

$$t_1 - t_0 = \frac{1}{K} \cosh^{-1}\left(\frac{r_1}{r_0}\right). \tag{52}$$

3.2.2 Determination of t_1

Equation (43) can be rewritten as:

$$\vec{v} = \begin{pmatrix} \dot{r}\cos(\omega t + \zeta) - r(\omega + \dot{\zeta})\sin(\omega t + \zeta) \\ \dot{r}\sin(\omega t + \zeta) + r(\omega + \dot{\zeta})\cos(\omega t + \zeta) \end{pmatrix} = \begin{pmatrix} v_H \\ v_V \end{pmatrix}. \tag{53}$$

At the exit, $t = t_1$, the two components of the velocity vector have to satisfy the condition expressed in Equation (50). We must have $v_V = 0$ or,

$$\dot{r}_1\sin(\omega t_1 + \zeta_1) + r_1(\omega + \dot{\zeta}_1)\cos(\omega t_1 + \zeta_1) = 0, \tag{54}$$

where the subscript "1" means that the quantity is taken at the exit. This equation can also be written as:

$$\dot{r}_1\tan(\omega t_1 + \zeta_1) + r_1\omega + r_1\frac{\dot{r}_1}{ar_1} = 0. \tag{55}$$

Then,

$$\omega t_1 + \zeta_1 = \arctan\left[-\frac{1}{a} - \frac{r_1}{\dot{r}_1}\omega\right] + k\pi, \tag{56}$$

where $\dot{r}_1$ can be obtained from Equation (47) and ζ_1 from Equation (19) where we replace r by r_1. Finally, we can write the expression of t_1 as a function of the parameters of the system:

$$t_1 = \frac{1}{\omega}\left(\arctan\left[-\frac{1}{a} - \frac{r_1\omega}{K\sqrt{r_1^2 - r_0^2}}\right] + k\pi - \frac{1}{a}\ln\frac{r_1}{r_0}\right). \tag{57}$$

The parameter k is adjusted so that the results are physically acceptable. The value of t_1 must be greater than $(t_1 - t_0)$ found previously. We can check the accuracy of the results here by plugging the expression of t_1 above

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into the first component of the velocity vector at the exit, v_H and it should satisfy the equation $v_H = v_{exit}$.

These calculations can now be applied to the practical case we are considering and the actual values of t_0 and t_1 can be determined. This will allow us to have a good picture of the launching process.

3.3 APPLICATION

The practical case considered is as follows:

- The radius of the rotor is: $r_1 = 0.1$ m.

- The distance from the center of the rotor at which the ball starts the acceleration process is: $r_0 = 0.01$ m.

- The track shape constant is taken as $a = 1$.

- The launcher is rotating at an angular velocity $\omega = 10000$ rad/s, which gives a tip speed of $v_{tip} = 1$ km/s and a revolution period of $T = 2\pi / \omega = 0.63$ ms.

The results are as follows:

1. The ball leaves the launcher with an exit velocity $v_{exit} = 1.7$ km/s.

2. The timing is such that:

    - $t_1 - t_0 = 0.5$ ms from Equation (52),

    - $t_1 = 0.9$ ms from Equation (57), where $k = 4$,

    - and $t_0 = 0.4$ ms.

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The launch process for one ball lasts 0.9 ms, which is about one and a half revolutions of the rotor. For the first 0.4 ms of the launch process, the ball is kept near the axis of the rotor at the distance r_0, then it is released. After another 0.5 ms, it will roll off the track horizontally at a velocity of 1.7 km/s. We also substituted for t_1 in v_H at the exit using the numerical values above to check their consistency and we found $v_H = v_{exit} = 1.7$ km/s, which confirms the accuracy of the results.

The launch process is illustrated in Figure - 14 through Figure - 16 where we display the position of the track at different sequences of the process.

The last part of this launch process study consists of the roll versus skid analysis of the ball on the track. This will be the subject of the final section of this chapter.

3.4 ROLL VERSUS SKID ANALYSIS

The fundamental assumption for this system is that it is conservative. However, the high speed at which the ball strikes the track on entry and the acceleration at which it is flung outwards may cause the ball to skid on the track. Skidding would cause frictional losses making the system non-conservative. S. Pang considered this and drew some conclusions about the sliding tendency and the system parameters [29]. Skidding is considered here in the context of a high vacuum environment where the coefficients of friction are largest, as illustrated in Table - 3 [29]. A higher coefficient of friction between the ball and the track provides a higher tangential force, raising the critical point at which the ball begins to skid. However, if skidding indeed occurs, the frictional losses are severe.

The purpose of this analysis is to investigate the conditions that cause skidding and to determine design parameters to avoid skidding. In addition

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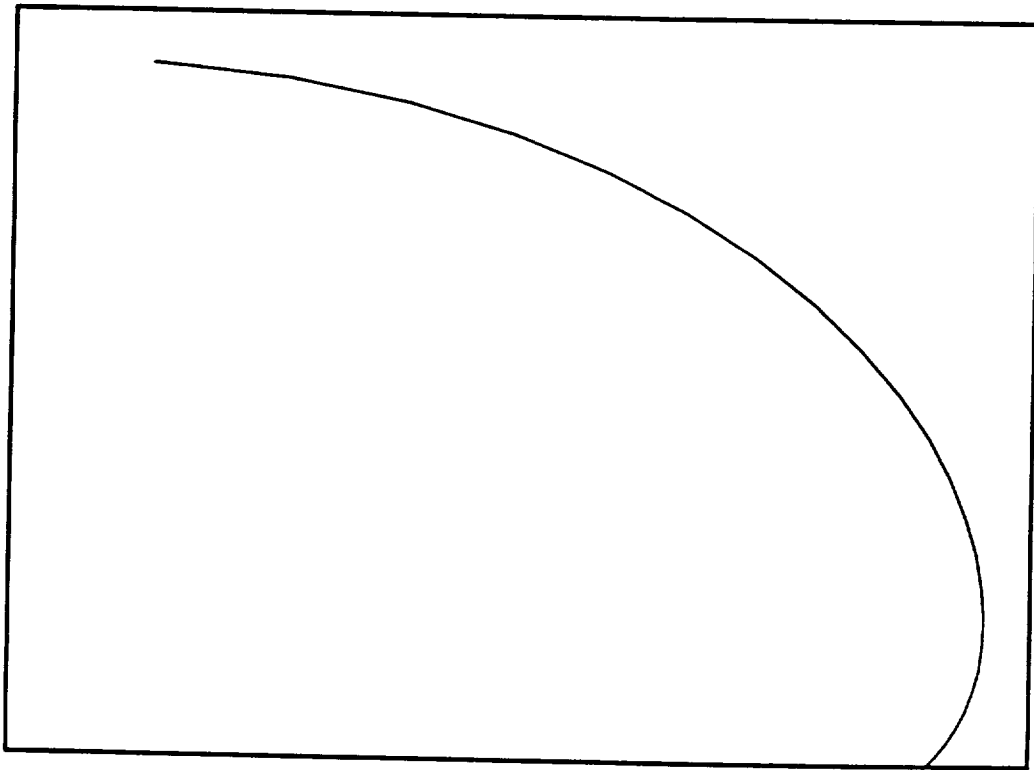
Figure - 14. Track orientation at the instant $t = 0$.
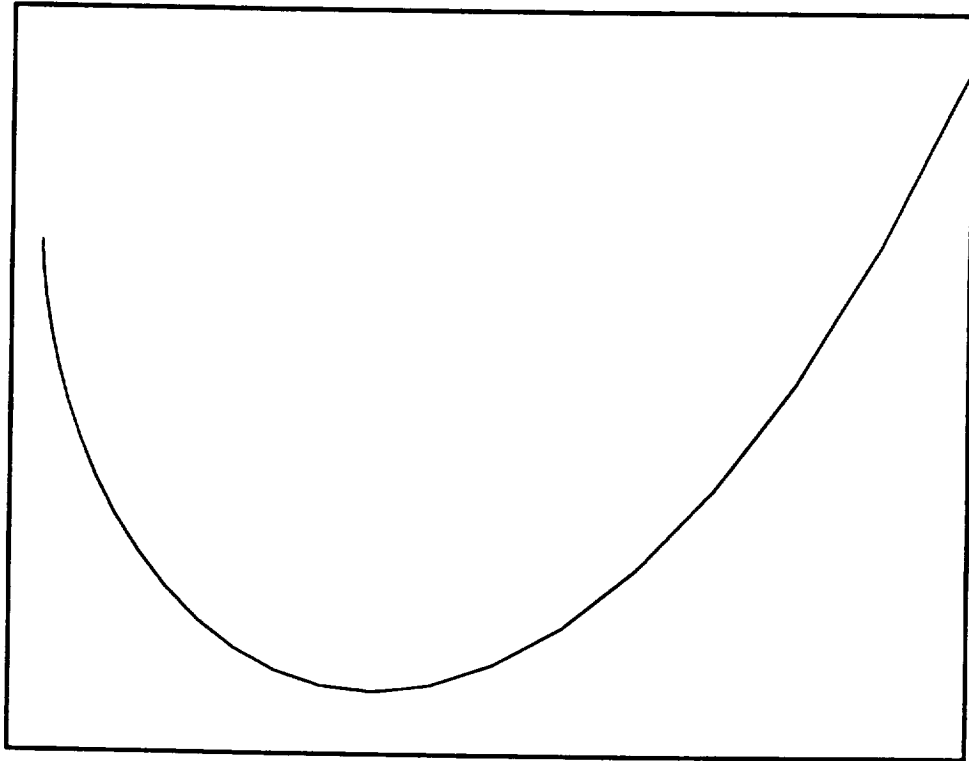
Figure - 15. Track orientation at the instant $t = t_0 = 0.4$ ms.
The ball is now released near the center of the lanucher.

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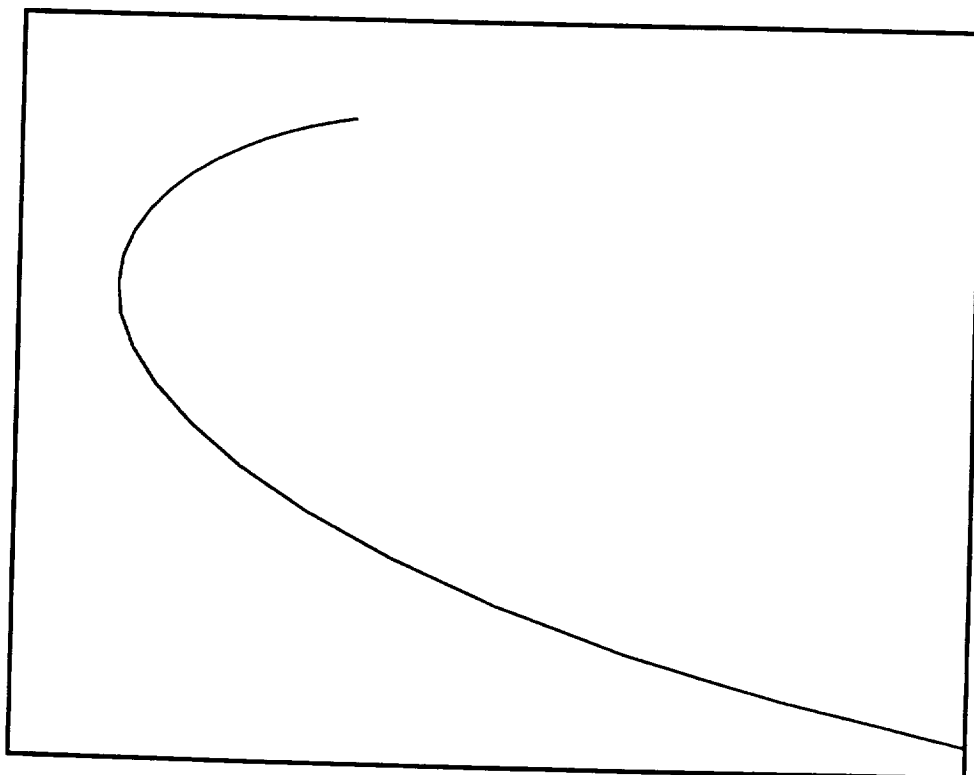
Figure - 16. Track orientation at the instant $t = t_1 = 0.9$ ms.
The ball now rolls of the track.

Material Pair	In Air	In High Vacuum
Al–Al	0.78	1.57
Be-Cu–Be-Cu	0.58	1.10
Brass–Brass	0.32	0.71
Copper–Copper	1.04	2.00
Be-Cu–Brass	0.38	0.9
Cu–Steel	0.55	0.96
Cr–Cr	0.85	1.30
Stainless Steel- -Stailess Steel	0.51	0.93

Table - 3. Table of coefficients of friction

to the normal force λ_1 restricting the ball to move in a spiral, a tangential force λ_2 constraining the ball to roll on the track is taken into account, as explained in the beginning of this chapter. The frictional force available between the two surfaces is $\mu\lambda_1$, μ being the coefficient of friction. If this available friction is less than the required friction λ_2, skidding will occur. So, to check for skidding, we need to check if $\lambda_2 > \mu\lambda_1$ or if the ratio λ_2/λ_1 is greater than μ. The methodology followed here is as described in [29]. To determine λ_2 and λ_1 we just go back to the beginning of this chapter and solve Lagrange's equations for these two forces.

The results are:

$$\lambda_2 = \left(\sqrt{1 + \frac{1}{a^2}}\right) \frac{\mathcal{I}}{\ell^2}\ddot{r},$$

from Equation (31). So, the tangential force per unit mass acting on the ball can be written as:

$$\frac{\lambda_2}{m} = \frac{2}{7}\left(\frac{a\omega^2}{\sqrt{1+a^2}}\right) r_0 \cosh K(t - t_0), \tag{58}$$

using Equations (13, 38) and the expression of $r(t)$ from Equation (41). Now, from Equation (32) combined with Equations (33, 34), we can write the expression of the normal force per unit mass acting on the ball as:

$$\frac{\lambda_1}{m} = \frac{\ddot{r}}{a^2} + \frac{\dot{r}^2}{a^2 r} + \frac{2\omega}{a}\dot{r}.$$

Again from the expression of $r(t)$ in Equation (41), λ_1/m becomes:

$$\frac{\lambda_1}{m} = \frac{5}{7} \frac{r_0 \omega^2}{1 + a^2} \left[\cosh K(t - t_0) + \sinh K(t - t_0) \tanh K(t - t_0) + \right.$$

$$\left. 2\sqrt{\frac{7}{5}}\sqrt{1 + a^2} \sinh K(t - t_0) \right] . \qquad (59)$$

Now, we can write the ratio of the tangential force to the normal force as:

$$\frac{\lambda_2}{\lambda_1} = \frac{2a\sqrt{1 + a^2}}{5\left[1 + \tanh^2 K(t - t_0)\right] + 2\sqrt{35}\sqrt{1 + a^2} \tanh K(t - t_0)} . \qquad (60)$$

Several calculations were done to see how the design parameters affect the sliding tendency of the ball. We can see that the mass and the radius of the ball have no effect on the sliding tendency. We can also see that, as the ball leaves the center (when $t = t_0$), the ratio is independent of K, since $\tanh K(t - t_0) = 0$. Therefore, the sliding tendency due to the choice of ω (or K), is maximum at the end of the track and not at the beginning. At that instant, $t - t_0 = 0.5$ ms, as determined previously in the practical case considered. So, we plotted λ_2/λ_1 versus ω at the exit of the track. This is illustrated in Figure - 17 and we can see that the ratio is practically constant. This can be understood from the fact that the Hyperbolic tangent varies between 0 and 1, and for the values of ω that we are interested in (around 10,000 rad/s), varying ω does not make the hyperbolic tangent vary much. The denominator in Equation (60) is then almost contant making the ratio independent of K, which, in turns is proportional to the angular velocity ω. This implies the irrelevency of ω as a design parameter in this analysis. Figure - 17 illustrates the ratio λ_2/λ_1 versus ω for 4 different values of the
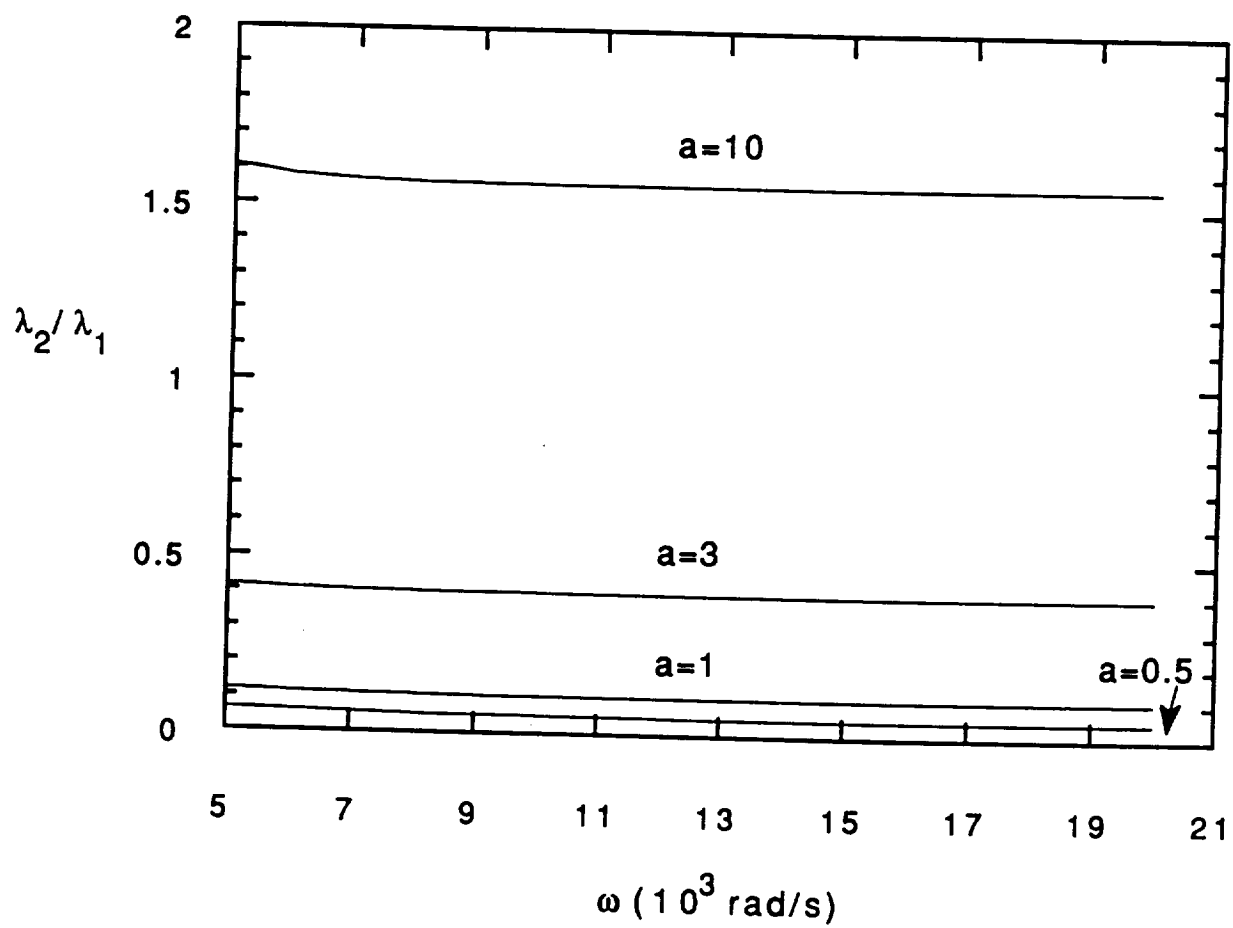
Figure - 17. Required coefficient of friction versus
the angular velocity of the rotor.

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track shape constant a, and we can see that the sliding tendency increases with increasing a. So, the design parameter affecting the sliding tendency is the track shape. We then maximize the ratio as a function of time to see how the geometry affects the sliding tendency in the worst case. We know that the argument $K(t - t_0)$ is positive. This makes $\tanh K(t - t_0)$ vary between 0 and 1, and then, the denominator in Equation (60) between 5 and $(10 + 2\sqrt{35}\sqrt{1 + a^2})$. Consequently, the ratio is such that:

$$\left(\frac{\lambda_2}{\lambda_1}\right)_{min} = \frac{2a\sqrt{1 + a^2}}{10 + 2\sqrt{35}\sqrt{1 + a^2}} \leq \frac{\lambda_2}{\lambda_1} \leq \frac{2a\sqrt{1 + a^2}}{5} = \left(\frac{\lambda_2}{\lambda_1}\right)_{max}.$$

And finally, we give in Figure - 18 an illustration of the relationship between the maximized sliding tendency $(\lambda_2/\lambda_1)_{max}$ and the track shape constant a. Flatter track produces a greater tendency to skidding. Once the materials are chosen, μ is fixed and the track shape constant a can be chosen not to exeed a critical value. It has to be such that λ_2/λ_1 (required coefficient of friction) is not greater than μ (available coefficient of friction). Otherwise, skidding will occur.

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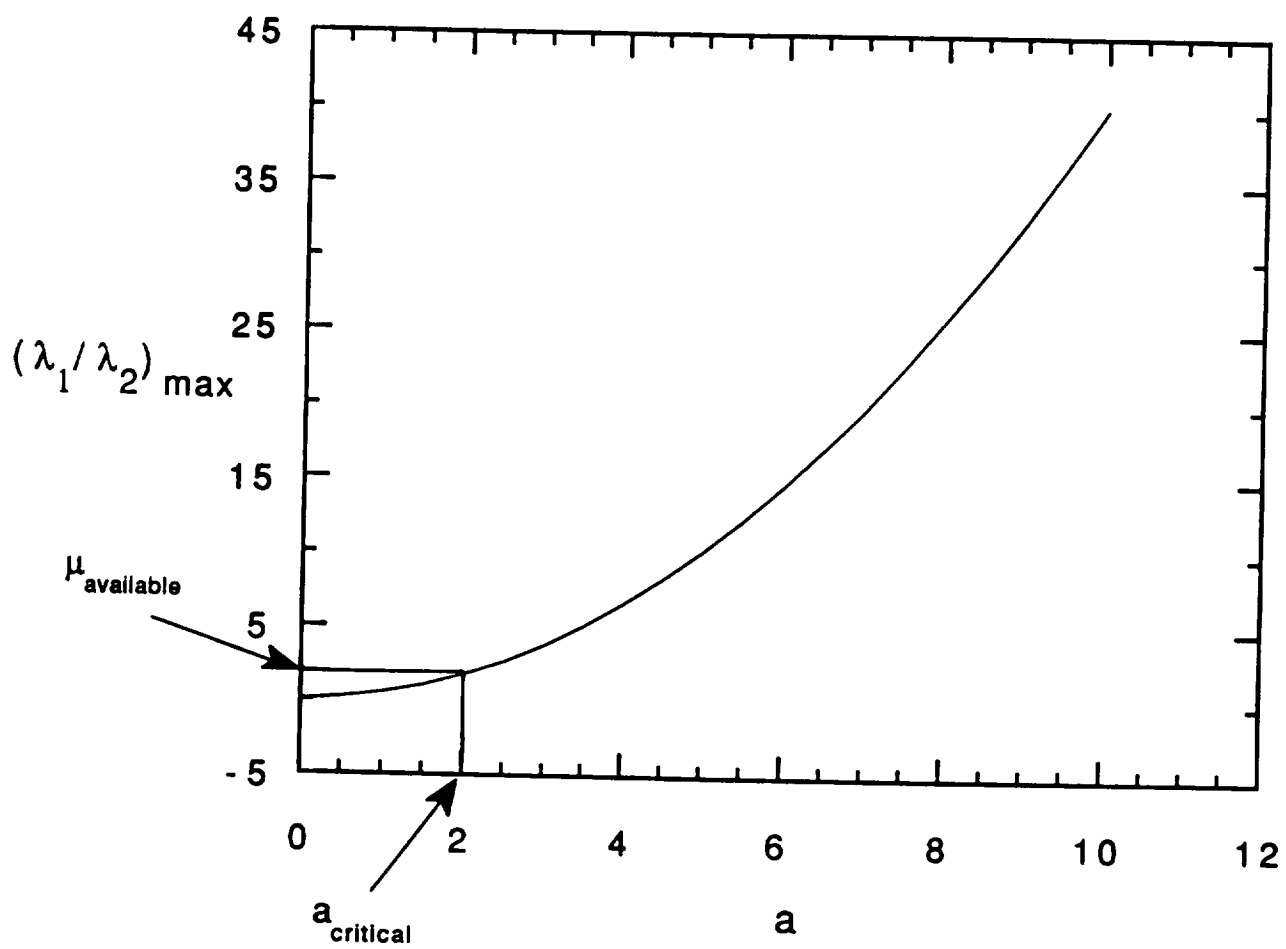
Figure - 18. Required coefficient of friction versus
the track shape constant.

Chapter 4

ANALYSIS OF THE CATCHING PROCESS

The ball, launched from the primary centrifuge according to the process described in detail in the previous chapter, is then caught on the second centrifuge. The resulting momentum transfer is used to propel this centrifuge.

4.1 CATCHING CONDITION

For a maximum momentum exchange, the velocity vectors of the points of impact on the ball and the rotating blade must be equal to insure no bouncing, nor sliding of the ball on the track. The normal components of the two velocity vectors have to match so that the ball does not bounce off the track, and the tangential components have to be equal to insure no sliding of the ball on the track. This catching condition can be summarized by the following equation:

$$\vec{v_1} - \vec{v_2} = 0, \tag{61}$$

where, v_1 is the velocity of the tip of the ball, and v_2 the velocity of the impact point on the track where the ball is to land. Solving Equation (61) will allow us to determine the exact position and timing for catching.

4.2 DETERMINATION OF CATCHING POSITION AND TIME

At the exit of the primary launcher, the center of mass of the ball has a linear velocity v_{exit}, as mentioned in the previous chapter. The ball is also rotating at a rate $\dot{\alpha}$ as explained before. This makes the impact point on the ball have a velocity $\vec{v_1}$ such that:

$$\vec{v_1} = \begin{pmatrix} v_{exit} - u\cos\xi \\ -u\sin\xi \end{pmatrix}, \tag{62}$$

where the angle ξ is defined in Figure - 18 and u is the rotating speed,

$$u = \dot{\alpha} \, |_{exit} \, \ell = \left(\sqrt{1 + \frac{1}{a^2}} \right) \dot{r} \, |_{exit},$$

from Equation (24) in the previous chapter, differentiated once with respect to time. This can be, equivalently written as:

$$u = \left(\sqrt{1 + \frac{1}{a_2}} \right) \dot{r}_1. \tag{63}$$

Substituting for $\dot{r}_1$ from Equation (47), also in the previous chapter, taken at the exit of the track, we can write:

$$u = \sqrt{\frac{5}{7}} \, \omega \, \sqrt{r_1^2 - r_0^2}. \tag{64}$$

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The velocity of the point of impact on the rotating track is:

$$\vec{v_2} = \begin{pmatrix} v_{relay} - r_0 \omega e^{a\zeta} \sin(\omega t + \zeta) \\ r_0 \omega e^{a\zeta} \cos(\omega t + \zeta) \end{pmatrix}, \tag{65}$$

where v_{relay} is the linear velocity at which the relay is moving. This velocity vector is obtained by differentiating with respect to time the position vector of the impact point on the blade,

$$\vec{r} = \begin{pmatrix} v_{relay} t + r_0 e^{a\zeta} \cos(\omega t + \zeta) \\ r_0 e^{a\zeta} \sin(\omega t + \zeta) \end{pmatrix}. \tag{66}$$

We have to keep in mind while differentiating that we are only interested in one point, the impact point. This means that ζ is not a variable here. It has a specific value that we have to solve for ζ_i.

From Equations (61, 62, 64) we get:

$$v_{relative} - u \cos \xi = -\omega r_0 e^{a\zeta_i} \sin(\omega t_i + \zeta_i), \tag{67}$$

and

$$- u \sin \xi = \omega r_0 e^{a\zeta_i} \cos(\omega t_i + \zeta_i), \tag{68}$$

where $v_{relative}$ is the relative speed of the two moving bodies (ball and relay),

$$v_{relative} = v_{exit} - v_{relay}, \tag{69}$$

and t_i is the time at which the ball impacts the track.

When we solve Equations (67, 68) the catching parameters, position on the blade, ζ_i and time at which catching occurs, t_i will be determined.

From Figure - 19, we can see that:

$$\tan \xi = \frac{dy}{dx} = \frac{dy/d\zeta}{dx/d\zeta},$$

where

$$x = r_0 e^{a\zeta} \cos(\zeta + \omega t),$$

and

$$y = r_0 e^{a\zeta} \sin(\zeta + \omega t).$$

Then,

$$\tan \xi = \frac{1 + a \tan(\zeta + \omega t)}{a - \tan(\zeta + \omega t)}. \tag{70}$$

Let us now consider the following identities:

$$\cos \xi = \pm \frac{1}{\sqrt{1 + \tan^2 \xi}},$$

and

$$\sin \xi = \pm \frac{1}{\sqrt{1 + \cot^2 \xi}}.$$

Using the relationship between $(\tan \xi)$ and $(\tan(\zeta + \omega t))$ expressed in Equation (70), $(\cos \xi)$ and $(\sin \xi)$ can be rewritten as:

$$\cos \xi = \pm \frac{(a - \tan(\zeta + \omega t))}{\sqrt{(1 + a^2)(1 + \tan^2(\zeta + \omega t))}},$$

and

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Track: $r = r_0\, e^{a\zeta}$
$a = -1$
$0 < \zeta < -2.3 \text{ rad}$

Figure - 19. Catching relay and track.

$$\sin \xi = \pm \frac{(1 + a\tan(\zeta + \omega t))}{\sqrt{(1 + a^2)(1 + \tan^2(\zeta + \omega t))}}.$$

Now, substituting for $(\cos \xi)$ and $(\sin \xi)$ as above in Equations (67, 68), we get:

$$v_{relative} \mp \frac{u(a - \tan(\zeta_i + \omega t_i))}{\sqrt{(1 + a^2)(1 + \tan^2(\zeta_i + \omega t_i))}} = -r_0 \omega e^{a\zeta_i} \sin(\omega t_i + \zeta_i), \quad (71)$$

and

$$\mp \frac{u(1 + a\tan(\zeta_i + \omega t_i))}{\sqrt{(1 + a^2)(1 + \tan^2(\zeta_i + \omega t_i))}} = r_0 \omega e^{a\zeta_i} \cos(\omega t_i + \zeta_i). \quad (72)$$

Dividing Equation (71) by Equation (72) we get:

$$\mp \frac{v_{relative}\sqrt{(1 + a^2)(1 + \tan^2(\zeta_i + \omega t_i))}}{u(1 + a\tan(\zeta_i + \omega t_i))} + \frac{a - \tan(\zeta_i + \omega t_i)}{1 + a\tan(\zeta_i + \omega t_i)}$$

$$= -\tan(\zeta_i + \omega t_i). \quad (73)$$

This can also be written as:

$$\mp \frac{v_{relative}}{u} \frac{\sqrt{(1 + a^2)(1 + \tan^2(\zeta_i + \omega t_i))}}{1 + a\tan(\zeta_i + \omega t_i)} = -\frac{a(1 + \tan^2(\zeta_i + \omega t_i))}{1 + a\tan(\zeta_i + \omega t_i)},$$

or simply,

$$\mp \frac{-v_{relative}}{u} \frac{\sqrt{1 + a^2}}{a} = \sqrt{1 + \tan^2(\zeta_i + \omega t_i)}. \quad (74)$$

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If we square this equation, we can solve for $\tan(\zeta_i + \omega t_i)$ as:

$$\tan^2(\zeta_i + \omega t_i) = \left(\frac{v_{relative}}{u}\right)^2 \left(1 + \frac{1}{a^2}\right) - 1. \qquad (75)$$

Now, we can go back to Equation (72) and rewrite it using the identity:

$$\cos(\zeta_i + \omega t_i) = \pm\frac{1}{\sqrt{1 + \tan^2(\zeta_i + \omega t_i)}}.$$

The equation then becomes:

$$-\frac{u(1 + a\tan(\zeta_i + \omega t_i))}{\sqrt{1 + a^2}} = r_0 \omega e^{a\zeta_i}, \qquad (76)$$

or, if we want to solve for ζ_i,

$$\zeta_i = \frac{1}{a}\ln\left[-\frac{u(1 + a\tan(\zeta_i + \omega t_i))}{r_0 \omega\sqrt{1 + a^2}}\right], \qquad (77)$$

where, $\tan(\zeta_i + \omega t_i)$ is given by Equation (75). The solution, ζ_i, gives the position of the ball on the track at catching. We can also solve for the argument $(\omega t_i + \zeta_i)$ of the tangent from Equation (75), and finally deduce t_i, the time at which catching must occur. This time can be written as:

$$t_i = \frac{1}{\omega}\left[\arctan\left(\sqrt{\left(\frac{v_{relative}}{u}\right)^2\left(1 + \frac{1}{a^2}\right) - 1}\right) - \zeta_i + k\pi\right], \qquad (78)$$

where, k is chosen so that the catching time is greater that the time the ball exits the launcher.

4.3 NUMERICAL APPLICATION

Let us now consider again the practical system proposed in the previous chapter and determine the position and time of catching. Let us assume at this point that the second centrifuge is not moving yet. It is just rotating at the same angular rate ω as the primary launcher. Consequently,

$$v_{relative} = v_{exit} = 1.7 \text{ km/s.}$$

We can also have from the previous chapter (Equation (47)) the value of $\dot{r}$ at the exit, which is, $\dot{r}_1 = 595$ m/s for the particular example analyzed in the previous chapter. Substituting for $\dot{r}_1$ in Equation (63), we can determine the value of u as:

$$u = 841 \text{ m/s.}$$

We also have:

- $r_0 = 0.01$ m,

- and $\omega = 10000$ rad/s, as before.

- The track shape is the same as for the launcher case but its orientation is different, and so the constant has a different value. For the catcher, the track shape constant is $a = -1$.

After substituting for $u, v_{relative}, r_0, \omega$, and a in Equation (75) we get:

$$\tan^2(\zeta_i + \omega t_i) = 7.17,$$

or,

$$\tan(\zeta_i + \omega t_i) = \pm 2.68.$$

Now, substituting this result in Equation (77), we fing the catching position:

$$\zeta_i = -2.3 \text{ rad},$$

which represents the edge of the track. This result confirms our intuitive feeling that the ball had to be caught on the edge of the track to satisfy the velocities matching as explained before. Finally, we can solve for t_i from Equation (78):

$$t_i = 1.3 \text{ ms},$$

where k was taken equal to 3.

Knowing the total time of the launch sequence $t_1 = 0.9$ ms from the previous chapter, we can deduce the time of flight of the ball between the two centrifuges. This time is:

$$t_{flight} = t_i - t_1 = 0.4 \text{ ms}.$$

And knowing the velocity at which the ball is travelling, we can determine what distance, d, initially we should place the relay from the primary launcher:

$$d = v_{exit} \, t_{flight} = 0.68 \text{ m}.$$

It is better to start with an initial separation of about 500 m to insure that the time the ball spends on the relay is very small compared to the total time of flight of the ball for kinematics studies purposes as we shall see later. The ball will then be caught several revolution later than the time t_i calculated above. We have to choose k such that the time of flight will correspond to the desired distance. This fact will not be taken into account in this chapter. We will consider the minimal separation between the launcher and the relay of 0.68 m, but we will make the necessary changes in the generalization chapter for multiple balls launching.

To conclude this section on the catching conditions, we illustrate the orientation of the track both at $t = 0$ and at $t = t_i$ in Figure - 20 and Figure - 21.

4.4 BEHAVIOR OF THE BALL ON THE CATCHER

Now that the first ball is caught smoothly at the edge of the blade without any bouncing or sliding, we have to determine its trajectory on the track. As for the launch process, Lagrange equations with multipliers are used to analyze the behavior of the ball on the centrifuge. The same constraints as before are on the system. The first specifies the track shape and the second insures that the system is conservative by constraining the ball to roll on the track. Equations (10) through (21) from *Chapter 3* still apply, although the orientation of the angle ζ and the track shape constant a are different in this case. The reader should refer to Figure - 19 where an illustration of the track on the catching rotor was given.

Equation (21) from the previous chapter gives in this case:

$$ds = - \left(\sqrt{1 + \frac{1}{a^2}} \right) dr. \qquad (79)$$

This equation can be integrated and, along with the initial condition, $s = 0$ when $r = r_1$, it gives a relationship between the track length and the radial position of the ball:

$$s(t) = \left(\sqrt{1 + \frac{1}{a^2}} \right) (r_1 - r). \qquad (80)$$

Then, the second constraint equation becomes:

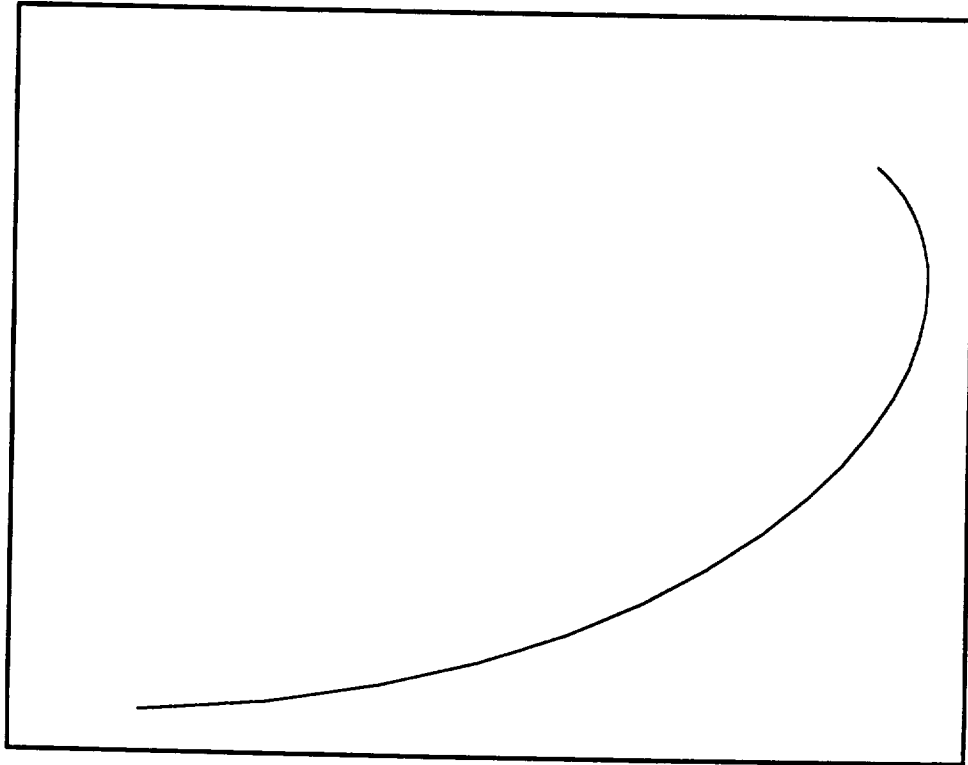$$f_2 = \ell \alpha(t) + \left(\sqrt{1 + \frac{1}{a^2}} \right) (r(t) - r_1) = 0. \qquad (81)$$

Figure - 20. Orientation of the track on the relay at the
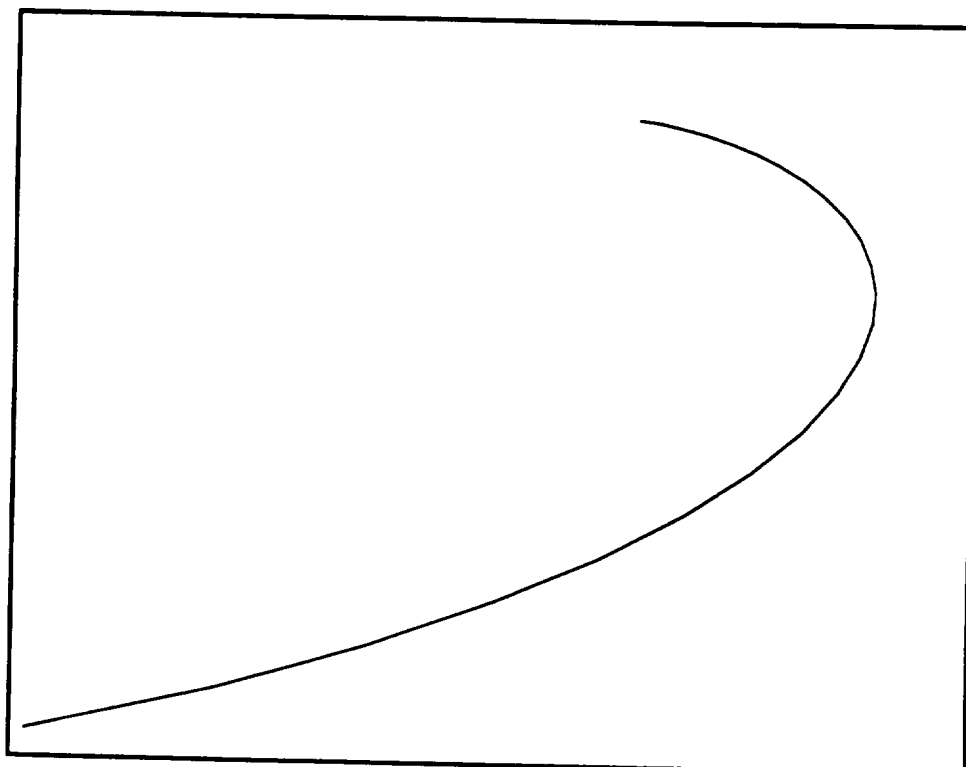instant $t = 0$.

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Figure - 21. Orientation of the track on the relay at the
instant $t = t_i = 1.3$ ms.
The ball now impacts the relay.

The equations of motion turned out to be exactly the same as for the launch process, and the governing equation is:

$$r(t) = A \cosh K(t - t_i) + B \sinh K(t - t_i). \tag{82}$$

The initial conditions at catching are known in terms of position and velocity:

$$r(t_i) = r_1,$$

and

$$v(t_i) = v_{exit},$$

for the center of mass.

Now, from $v(t_i)$ we can get $\dot{r}(t_i)$. We have:

$$v^2 = \dot{r}^2 + r^2(\omega + \dot{\zeta})^2,$$

from Equations (10, 11) of the previous chapter. We can also use the relationship between $\dot{\zeta}$ and $\dot{r}$ as expressed in Equation (33) in *Chapter 3* and the previous equation becomes:

$$v^2 = \dot{r}^2 \left(1 + \frac{1}{a^2} \right) + r^2\omega^2 + \frac{2}{a} r\dot{r}\omega, \tag{83}$$

or,

$$\dot{r}^2 \left(1 + \frac{1}{a^2} \right) + \frac{2}{a} r\dot{r}\omega + r^2\omega^2 - v^2 = 0.$$

The solution to this equation is:

$$\dot{r} = \frac{-\frac{2}{a}r\omega \pm \sqrt{D}}{2\left(1 + \frac{1}{a^2}\right)}, \tag{84}$$

where D is defined as:

$$D = 4\left[v^2 \left(1 + \frac{1}{a^2} \right) - r^2\omega^2 \right]. \tag{85}$$

Equation (84) can be expressed as:

$$\dot{r} = \frac{-ar\omega \pm a\sqrt{v^2(1+a^2) - a^2r^2\omega^2}}{1+a^2}.$$ (86)

At $t = t_i$ we get:

$$\dot{r}(t_i) = \frac{-ar_1\omega + a\sqrt{v_{exit}^2(1+a^2) - a^2r_1^2\omega^2}}{1+a^2},$$ (87)

where the "+" sign was chosen over the "−" sign because $\dot{r}(t_i)$ has to be negative since $r(t)$ is decreasing from r_1, radius of the rotor, to r_0 near the center. We also know that the track shape constant a is, in the case of the catcher, negative. It turns out that:

$$\dot{r}(t_i) = -\dot{r}_1,$$

where $\dot{r}_1$ is the value of $\dot{r}$ at the end of the launch sequence (at time t_1). Now, from Equation (82) we can write:

$$r(t_i) = A.$$

Differentiating Equation (82) with respect to time and setting $t = t_i$ we can also write:

$$\dot{r}(t_i) = BK.$$

Identifying these two equations with the initial conditions at catching as expressed before we get:

$$A = r_1,$$

and

$$B = -\frac{\dot{r}_1}{K}.$$

The trajectory of the ball on the catcher blade is given at all times by:

$$r(t) = r_1 \cosh K(t - t_i) - \frac{\dot{r}_1}{K} \sinh K(t - t_i). \tag{88}$$

Let us denote the time it takes for the ball to reach the center of the rotor by t_2. Solving Equation (88) at the center ($r(t) = r_0$) gives the value of the interval of time ($t_2 - t_i$). This interval of time is the time actually spent by the ball on the track of the catching rotor while t_2 is the total time from the very beginning of the launching process ($t = 0$).

The ball is now going to be transferred to another track on the catcher and then be propelled forward or backward. For the purpose of this partial study, we will only consider throwing it back to the primary launcher.

4.5 RETURN OF THE BALL TO THE PRIMARY LAUNCHER

The ball is now on the second track of the catcher to be propelled backward. The track shape is always the same and the orientation is opposite to the track where the ball is caught making it similar to the track where the ball is accelerated on the primary launcher. The trajectory of the ball is then known. We just need to replace t_0 in Equation (41) (*Chapter 3*) by t_3 and we can write:

$$r(t) = r_0 \cosh K(t - t_3), \tag{89}$$

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where t_3 is the time at which the ball leaves the center of the catcher and starts rolling back off toward the edge.

The position and the velocity of the ball as functions of time are described in Equations (42, 43) in the previous chapter. We also need to satisfy Equation (50) from that chapter to insure that the ball leaves exactly backward (horizontally).

Let t_4 denote the time at which the ball leaves the edge of the catcher. We can find the value of this time by solving the condition expressed in Equation (50) mentioned before. This condition has been developed in *Chapter 3*, we will then just use the result in Equation (57) (previous chapter), where we replace t_1 by t_4. The value of k is adjusted so that the results are physically acceptable. The value of t_4 has to be greater than $(t_2 + (t_4 - t_3)$). The value of $(t_4 - t_3)$ can be determined by solving Equation (89) at the edge where $t = t_4$ and $r(t) = r_1$.

Finally, $(t_3 - t_i)$, the transfer time between the two tracks and possibly some extra delay time where the ball is maintained near the rotor axis to insure proper orientation of the centrifuge, can now be deduced knowing $(t_4 - t_3)$, t_4, and t_i.

4.6 PRACTICAL CASE

This is a continuation of the practical case considered previously. Let us use the same specifications as before for the centrifuge:

- $r_1 = 0.1$ m,

- $\dot{r}_1 = 595$ m/s,

- $K = 5976$.

The trajectory of the ball on the track is described by:

$$r(t) = 0.1 \cosh K(t - t_i) - 0.099 \sinh K(t - t_i).$$

Figure - 22 shows the position of the ball as a function of time. $r(t)$ decreases from r_1 to r_0 in an interval of time $(t_2 - t_i) = 0.5$ ms.

Knowing t_i from the previous section, $t_i = 1.3$ ms, we can now deduce t_2. So, the total time for the ball to reach the center of the catcher from the time we start spinning the centrifuges is $t_2 = 1.8$ ms. The position of both tracks is represented at this time in Figure - 23 and Figure - 24.

We also illustrate $\dot{r}(t)$ versus t in Figure - 25. The mathematical expression for $\dot{r}(t)$ as a function of time is:

$$\dot{r}(t) = 597.6 \sinh K(t - t_i) - 595 \cosh K(t - t_i).$$

It starts at $-\dot{r}_1 = -595$ m/s and reaches the value -4.25 m/s after 0.5 ms which gives a velocity value of the center of mass, according to Equation (83), of 104.33 m/s at r_0 near the center of the rotor (about 6% of its initial value). This should help the transfer to the second track.

Now, before we can determine how long the ball is going to be kept near the center, we can calculate the interval of time $(t_4 - t_3)$ as explained before. We find:

$$t_4 - t_3 = 0.5 \text{ ms},$$

still the same time as before since the ball is rolling from r_0 at rest, to the edge, r_1.

The next step is to calculate t_4 knowing that it has to be greater than 2.3 ms. The result is:

$$t_4 = 2.48 \text{ ms},$$

with a value of k equal to 9.
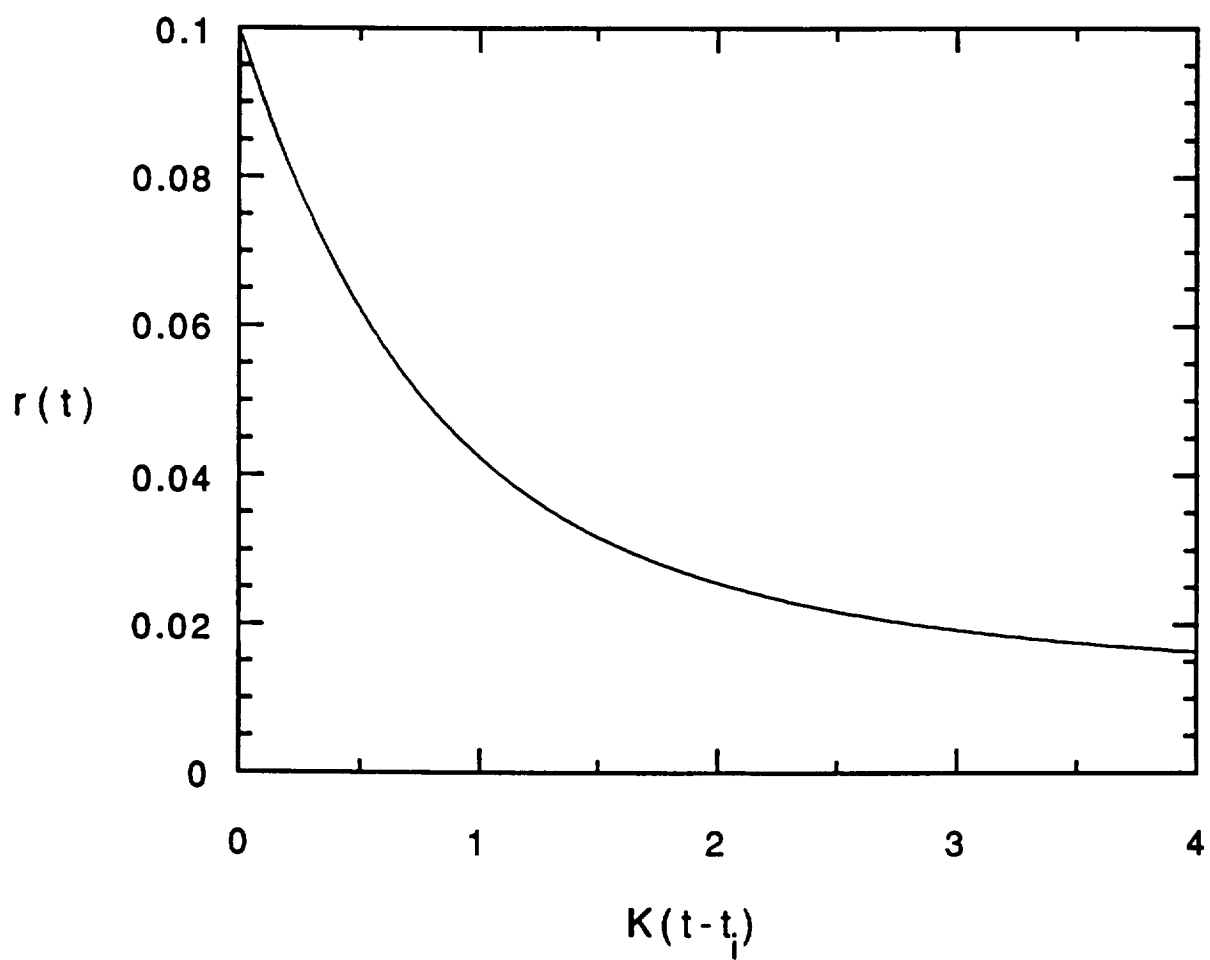
Figure - 22. Radial position of the ball on the relay.
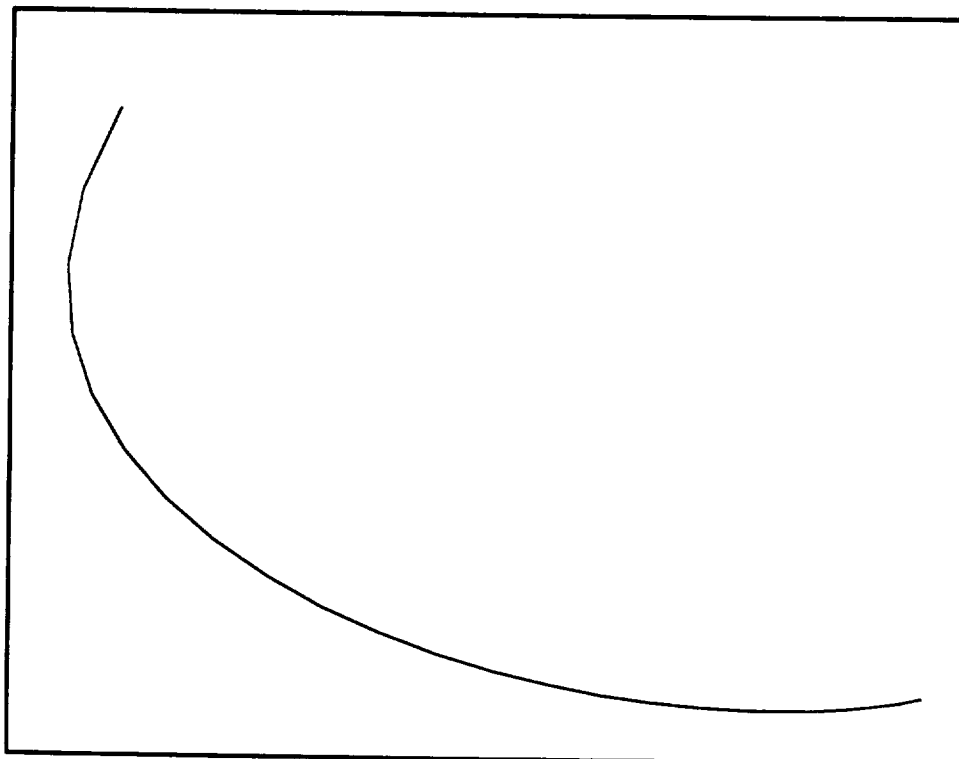
Figure - 23. Orientation of the track on the relay at the
instant $t = t_2 = 1.8$ ms.
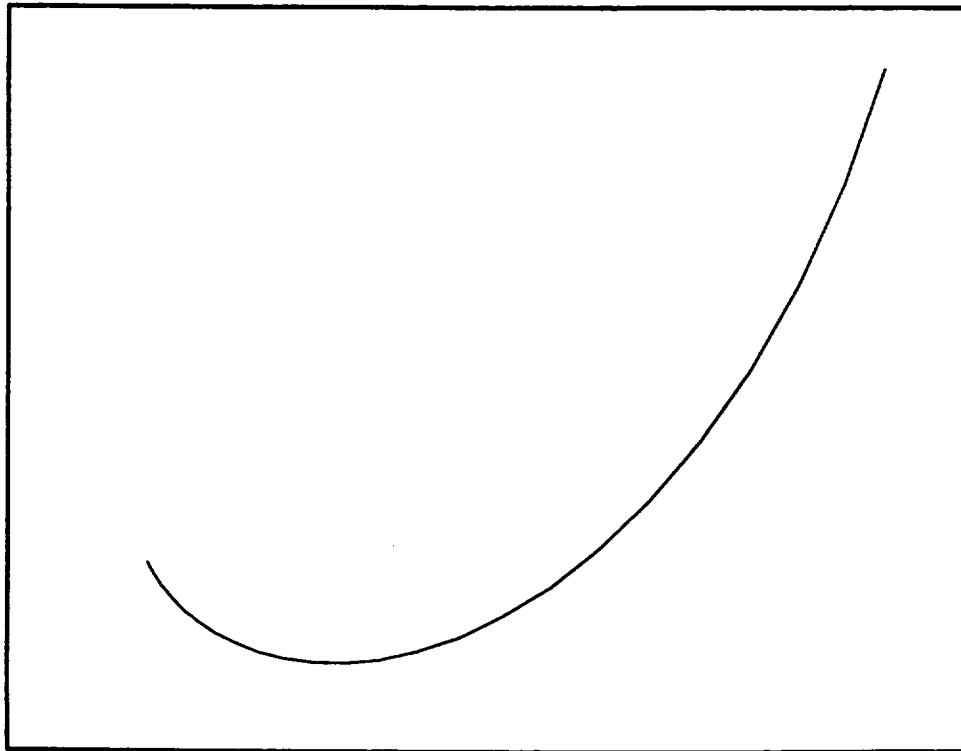The ball is now at the center of the relay.

Figure - 24. Orientation of the 2nd track on the relay
(acceleration part) at the instant $t = t_2 = 1.8$ ms.
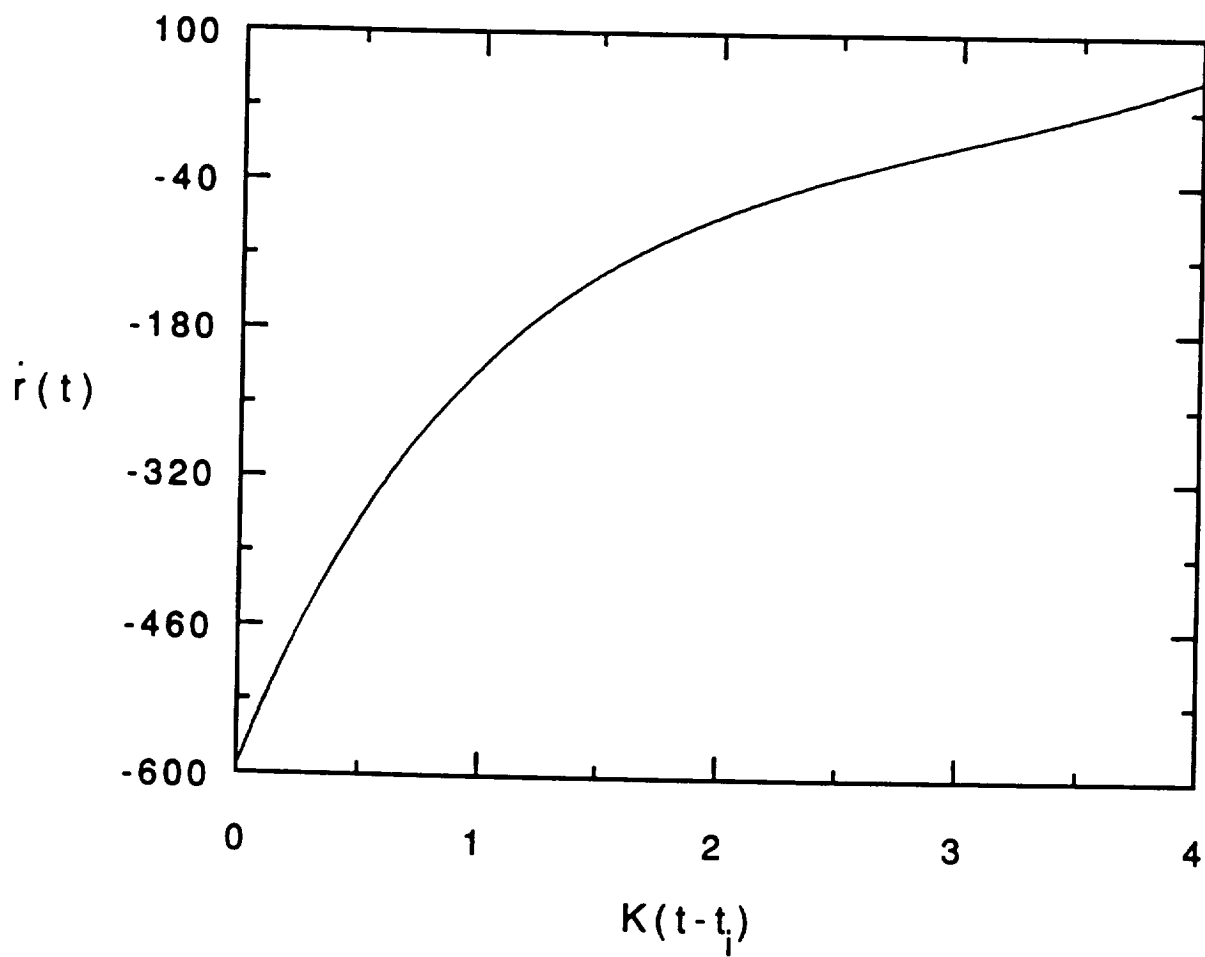The ball is now being transferred to this track.

Figure - 25. Variations of $\dot{r}(t)$ from the catching
point to the center of the relay.

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And finally, we can deduce t_3:

$$t_3 = 1.98 \text{ ms.}$$

From the instant t_2 to the instant t_3, for 0.18 ms, the ball will be transferred to the second track and will be kept there until the end of this time interval, when a new acceleration sequence begins. An illustration of the position of the track at this particular moment is given in Figure - 26. The ball is now released from the center.

The total time for the ball to be accelerated, launched, caught, and for it to roll back off the catcher to its edge is 2.48 ms. It leaves with a velocity of 1.7 km/s backward. An illustration of the rotor blade from where the ball is going to be launched backward at that particular time is given in Figure - 27.

Assuming that the catcher is still at the same distance from the primary launcher after this first ball catching, the time of flight between the two centrifuges is still 0.4 ms as before. This means that the total time it takes for the first ball to be launched and collected back by the primary launcher after it accomplishes its mission is 2.88 ms.

At this point, we can determine the momentum transferred by this first ball to the relay it impacted. We found here that $\vec{v}_{out} = -\vec{v}_{in}$, which means that the decrease in the ball velocity as defined in *Chapter 2*, becomes:

$$\Delta \vec{v} = 2\vec{v}_{in}, \tag{90}$$

and the momentum gained by the relay is:

$$M(\vec{V}_{out} - \vec{V}_{in}) = 2m\vec{v}_{in}. \tag{91}$$

In the particular case considered, the numerical value of this momentum gain is 14 kg m/s, if the ball weighs 4 grams (0.5 cm radius steel ball). Furthermore, if the relay is 10 kg, then the velocity increment is 1.4 m/s.
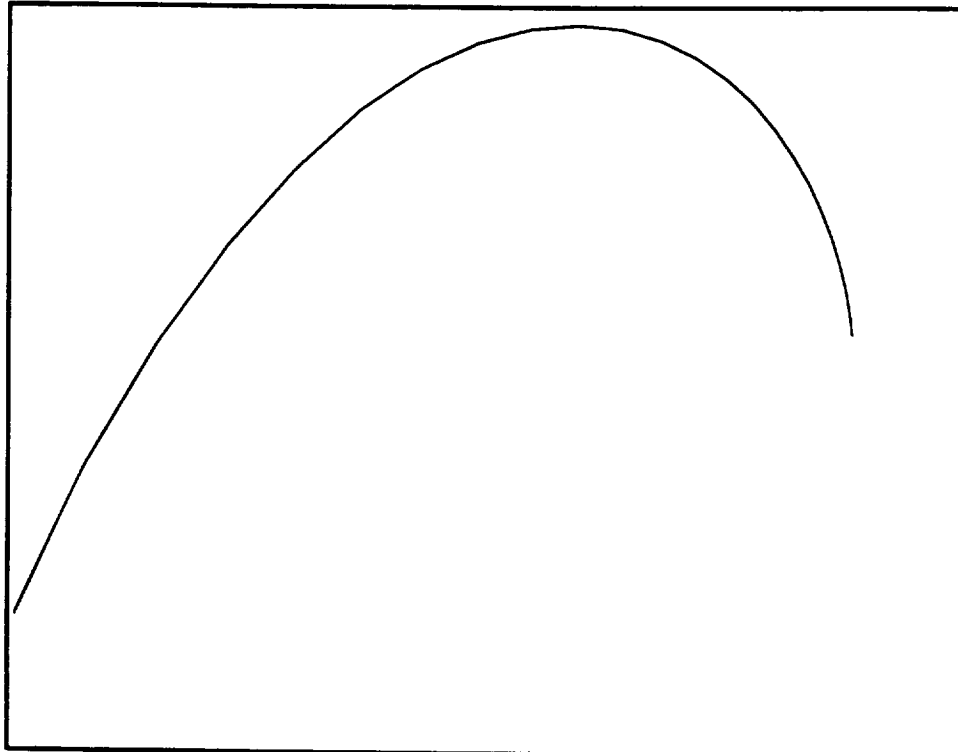
Figure - 26. Orientation of the 2nd track on the relay
at the instant $t = t_3 = 1.98$ ms.
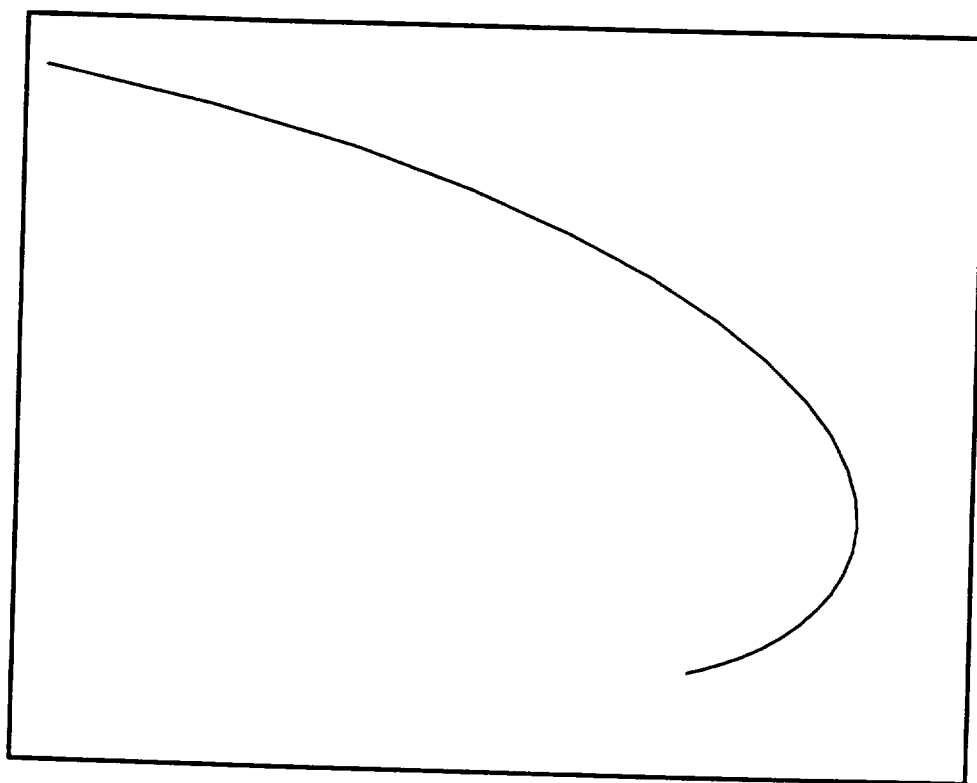The ball is now released from the center.

Figure - 27. Orientation of the 2nd track on the relay
at the instant $t = t_4 = 2.48$ ms.
The ball now flies off the relay.

Now, in terms of angular momentum, we have:

$$\vec{L}_{in} = -\vec{r}_1 \times m\vec{v}_{in}, \tag{92}$$

and

$$\vec{L}_{out} = \vec{r}_1 \times m(-\vec{v}_{in}). \tag{93}$$

This means that the angular momentum of the ball before and after the interaction with the relay is exactly the same and $\Delta \vec{L}_{rotor} = 0$. There is no angular momentum gained by the rotor after the launch of this first ball. This can be explained by the fact that the ball transfers all its angular momentum to the relay when it lands on the track and rolls up to the center of the rotor where it stops. So, in order for the ball to be accelerated on the second track and leave with the same velocity it impacted the relay the first time, the ball gets back the angular momentum it previously transferred to the relay. Therefore, the relay gains $\vec{L}_{in}$ in a first step, and then loses $\vec{L}_{out}$ in a second step, which is exactly the same quantity, leaving it with zero net angular momentum gain. This is not necessarily the case for the next balls launched. It depends on whether or not the ball is leaving with exactly the same angular momentum it had when it impacted the relay. This in turn, depends mainly on the position where the ball is caught on the track of the relay and where it exits this relay.

Chapter 5

GENERALIZATION: MULTIPLE LAUNCH ANALYSIS

After the detailed analysis of the launching and the catching of the first ball, as described in *Chapter 3* and *Chapter 4* respectively, we are now to consider multiple balls launching and catching, until our objective of accelerating the relay to the desired speed is achieved. We will start by generalizing the results obtained in *Chapter 3* to describe the acceleration of the nth ball on the primary launcher. We will then describe the catching position and time, and the behavior of the nth ball on the relay, based on our analysis in *Chapter 4*. We will also give the velocity increments the relay is getting from its impact with the nth ball. All these results are to be coded in a computer program that would keep track of all the information conserning the acceleration of the spaceborne relay.

5.1 LAUNCH OF n BALLS

We can determine at this point the frequency of launch of the projectiles. The maximum frequency would be just the same as the frequency of the rotational movement of the centrifuge, which is in the case of the application,

1.59 kHz. This means that we launch one ball per revolution. However, we will consider launching one ball every two revolutions instead, to make sure that the previous ball had enough time to leave the relay, when the newly launched ball impacts it. When the track is in the right position to launch the first ball, which is $t = 0.4$ ms in the particular case considered before, we wait for exactly two revolutions (1.26 ms) and release the second ball. The time would then be $t = 1.66$ ms. The first ball would have already left the launcher. It will also take this second ball 0.5 ms to get to the tip of the rotor and it will leave at $t = 2.16$ ms. We can generalize this result and state that soon after one ball leaves the launcher, another one is released near the center of that launcher. The exact value of this time delay between the exit at the tip of the rotor of one ball and the release of the next one, is determined by the difference between two periods of rotation of the centrifuge and the time it takes the ball to reach the tip of the rotor when it is accelerated from the center. The nth ball will be released near the center at a time $t_{release}$:

$$t_{release} = (0.4 + 1.26 \, (n - 1)) \text{ ms},$$

in the case of this application, or in general:

$$t_{release} = t_0 + 2 \, (n - 1) \, T, \tag{94}$$

and it will leave the track at

$$t_{exit} = (0.9 + 1.26 \, (n - 1)) \text{ ms},$$

in this particular case, which can also be generalized as:

$$t_{exit} = t_1 + 2 \, (n - 1) \, T \tag{95}$$

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5.2 VELOCITY OF THE RELAY AFTER THE LAUNCH OF THE nTH BALL

Based on our kinematics calculations in *Chapter 2*, the velocity of the relay after the launch of the nth ball would be given by Equation (8) in that chapter, where V_{in} would be the relay velocity after the $(n-1)$th ball leaves heading back to the primary launcher. We can just put that equation in a loop specifying the initial velocity of the relay, which is zero (starting from rest), and the program will give us the velocity of the relay after each ball leaves it.

5.3 DETERMINATION OF THE CATCHING POSITION AND TIME FOR THE nTH BALL

As we mentioned previously, we want an initial separation of about 500 m between the primary launcher and the relay. Therefore, when we code Equation (78) (*Chapter 4*) in the program, we keep on increasing the value of the integer k until the corresponding catching time, t_i, is such that the time of flight of the ball, t_{flight}, corresponds to a distance of at least 500 m. This will set the initial distance. Then we will solve Equation (78) mentioned above, to get the time at which the nth ball must be caught, and k would be chosen so that the result is physically acceptable. This means that the time of catching needs to be at least equal to the time the nth ball exits the launcher, plus the time of flight, which can be formulated mathematically as:

$$t_i > t_{exit} + t_{flight}$$

For the determination of the catching position, we just solve Equation (77) as explained before in *Chapter 4*, and we will find the angular position, ζ_i, of the ball on the track at the impact point. It is important to note here that

one condition has to be met when solving Equation (77). This condition is:

$$1 + a \tan(\zeta_i + \omega t_i) < 0. \tag{96}$$

This condition was not mentioned before in *Chapter 4* because it was satisfied in the case of the first ball, in the particular example considered. However, since we are generalizing the results in this chapter, we will have to check this condition and make sure it is satisfied. The first ball is caught at the edge of the track, but, as we will see later from the results of the computer program, the second ball will be caught slightly away from the edge, toward the center. Actually, the difference in angular position, ζ_i, of the second ball catching point and the first ball is very small (one thousandth of a radian). This difference between two successive balls is very small, but between the 1st and the nth, it will be quite large. The catching point will move farther and farther from the edge, until it becomes, eventually, near the center of the relay, when this one is moving at a speed comparable to the speed of the incoming balls. Therefore, $\tan(\zeta_i + \omega t_i)$ will change a great deal from the value calculated for the first ball, and we want to make sure that it always satisfies the condition expressed in Equation (96) above. Otherwise, catching cannot be achieved. The system of Equations (71,72) from the previous chapter, will not have a solution, and the catching condition of matching velocity vectors at the impact point would be impossible. It turns out, after we run our program, that this condition (Equation (96)), limits the number of balls launched to 882 in the case of the specific example treated. This represents actually a little more than 1/10th of the total number of balls needed to get the relay going at a velocity almost equal to the velocity of the incoming balls ($v_{relay} = 1.698$ km/s for $n = 8000$, and $v_{ball} = 1.7$ km/s). This number of balls will indeed be sufficient the accelerate the relay to more

than half the velocity of the balls. The relay velocity obtained with $n = 882$ is $v_{relay} = 861$ m/s. This is the limit of what we can achieve. A display of the catching position of the last ball is given in Figure - 28. We cannot possibly launch and catch more balls unless we change the parameters of the system, namely, the track shape. We tried a more circular track, with a constant $a = 0.5$, to improve the velocity of the relay, but the change would not be dramatic. The exit velocity of the balls would be in this case $v_{exit} = 1792$ m/s, and the maximum number of balls we can catch smoothly would be $n = 946$. The resulting relay velocity would then be $v_{relay} = 951$ m/s. A display of this track is given in Figure - 29. We chose to keep the previous case, $a = 1$, since the improvement is not very important compared to the trouble that we may have for mechanical design: machining a multiple tracks rotor as shown on Figure - 30.

5.4 BEHAVIOR OF THE nTH BALL ON THE RELAY

The nth ball will go through two different stages depending on which track it is on. At first, it will roll up the track from the catching position and stop at the center of the relay. Then, it will be transferred to the second track, where it will be accelerated and roll back toward the edge of the relay. It will fly off the track and go back to the primary launcher. We will now consider these two cases separately.

5.4.1 From Catching Position to Center of Relay

The first ball being caught at the edge of the relay, will roll for 0.5 ms to reach the center, which is the same as the time it takes the ball to reach the edge when accelerated from the center on the primary launcher. However,
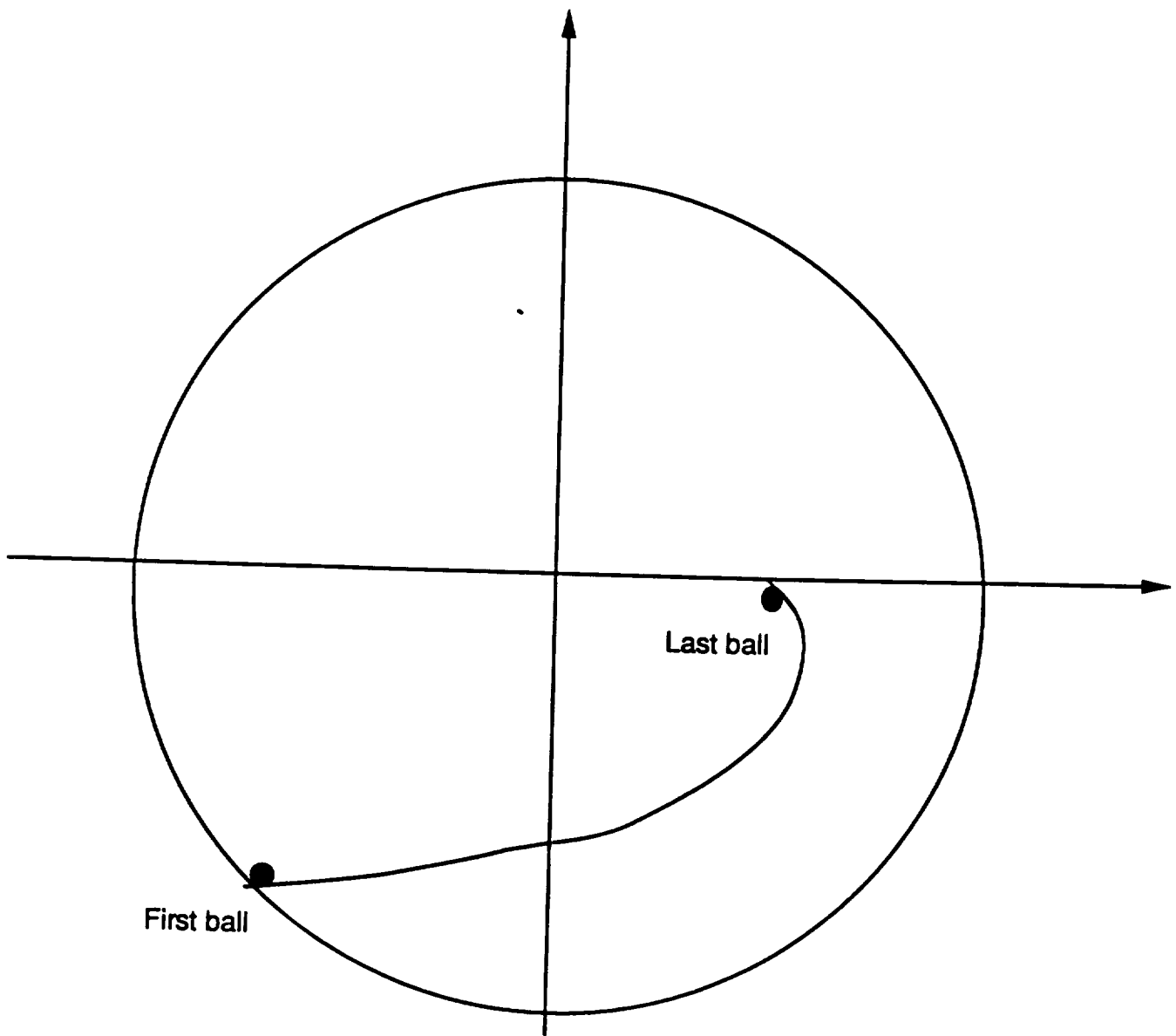
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Figure - 28. Catching position of the first and the last ball launched.
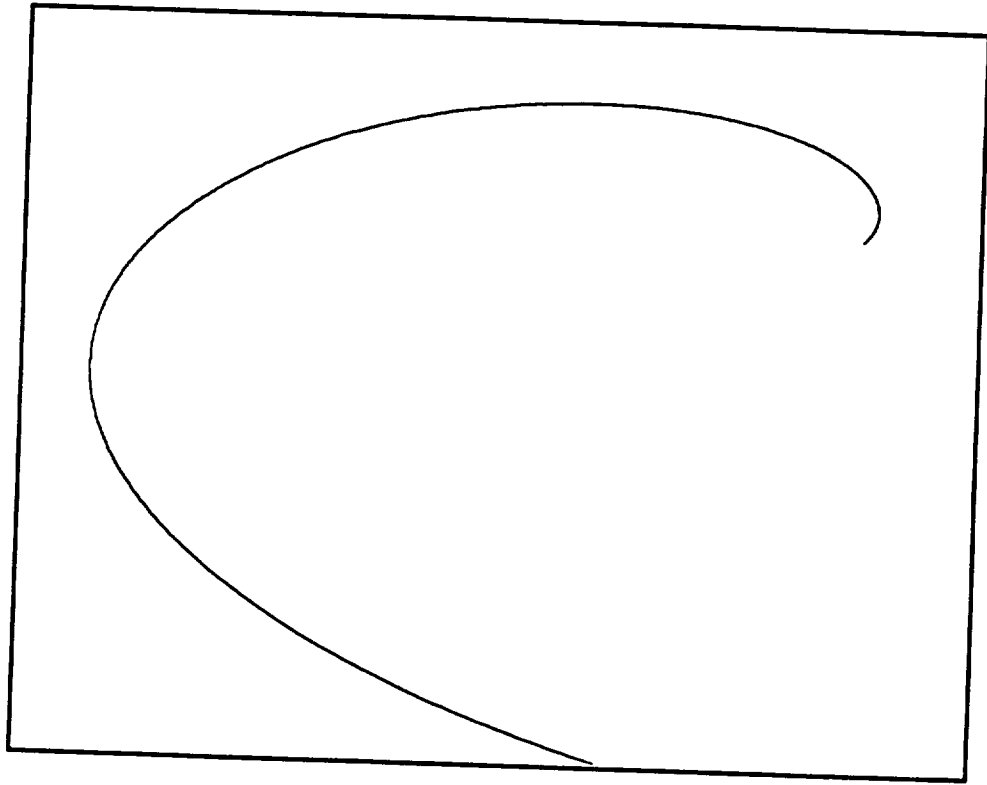
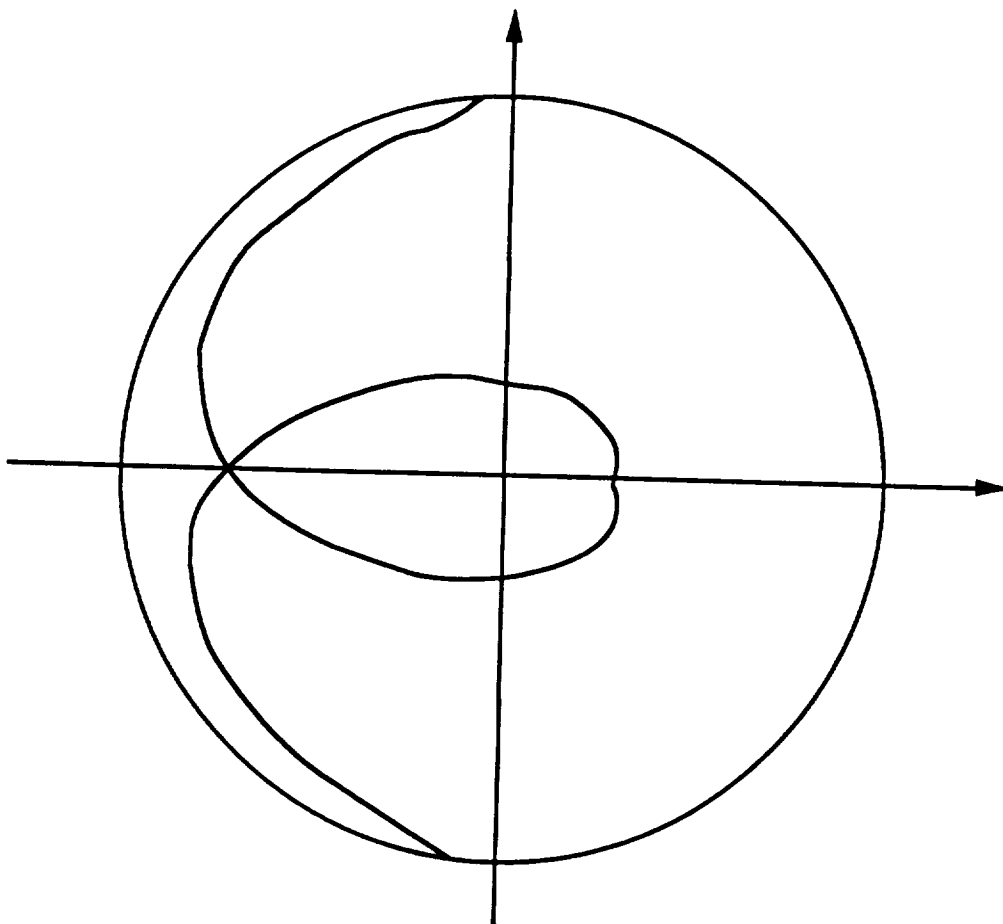Figure - 29. Logarithmic spiral track with a track shape constant $a = 0.5$.

Figure - 30. Multiple track relay with a track shape constant $a = 0.5$.

for the second ball, since it is caught a little farther from the edge, toward the center, the time needed for it to reach the center will be slightly shorter, and for the next ball, it will be even shorter, and so on, until the last ball launched. To find the exact value of this time, we need to solve Equation (88) in *Chapter 4*, where we replace r_1 by r_i, and $-\dot{r}_1$ by $\dot{r}_i$ for the general case of the nth ball. The equation becomes:

$$r(t) = r_i \cosh K(t - t_i) + \frac{\dot{r}_i}{K} \sinh K(t - t_i). \tag{97}$$

One way to solve this equation analytically would be to use the following identity:

$$\cosh^2 x - \sinh^2 x = 1,$$

or,

$$\sinh x = \pm\sqrt{\cosh^2 x - 1}.$$

Now, since the argument of the hyperbolic sine in our case, $K(t - t_i)$, t being the time we are solving for, is positive, $\sinh K(t - t_i)$ is also positive. Therefore, we will chose the "+" sign over the "-" sign in the expression above. Now substituting for $\sinh K(t - t_i)$ as explained above in Equation (88) mentioned earlier, we get:

$$r(t) = r_i \cosh K(t - t_i) + \frac{\dot{r}_i}{K}\sqrt{\cosh^2 K(t - t_i) - 1}. \tag{98}$$

Let us denote $\cosh K(t - t_i)$ by X, for simplification purposes. Then, Equation (98) can be written as:

$$r(t) = r_0 = r_i X + \frac{\dot{r}_i}{K}\sqrt{X^2 - 1}, \tag{99}$$

or,

$$r_0 - r_i X = \frac{\dot{r}_i}{K}\sqrt{X^2 - 1}. \tag{100}$$

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We can square both sides of Equation (100) and rewrite it as:

$$X^2 \left(\frac{\dot{r}_i^2}{K^2} - r_i^2 \right) + 2r_0 r_i X - \left(\frac{\dot{r}_i^2}{K^2} + r_0^2 \right) = 0. \tag{101}$$

The solution to this equation is:

$$X = \frac{-r_0 r_i \pm \sqrt{\Delta}}{\frac{\dot{r}_i^2}{K^2} - r_i^2}, \tag{102}$$

where Δ is given by:

$$\frac{\dot{r}_i^2}{K^2} \left[\frac{\dot{r}_i^2}{K^2} - (r_i^2 - r_0^2) \right].$$

We would note here that we have to chose the positive root only since X represents the hyperbolic cosine, which is always positive.

The next step is to solve for the argument, $K(t - t_i)$, of the hyperbolic cosine. Let us call this argument x. we can write:

$$X = \frac{e^x + e^{-x}}{2}.$$

If we denote e^x by Y, we can write the equation above as:

$$X = \frac{1}{2}(Y + \frac{1}{Y}),$$

or simply,

$$Y^2 - 2XY + 1 = 0. \tag{103}$$

The solutions to this equation can be written as:

$$Y = X \pm \sqrt{X^2 - 1}.$$

We have to keep in mind that Y has to be positive, since $Y = e^x$. We will chose the "+" sign in the above. One could argue that the root with the "-" sign is also positive. But, even when positive, it will be a relatively

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small number, which in turns means that the argument of the exponential, $x = K(t - t_i)$, would be negative. And this last statement is not physically acceptable. Therefore, we will chose the larger solution of the two, which is the one with the "+" sign.

We can finally write the time the needs the reach the center of the relay:

$$t - t_i = \frac{1}{K} \ln \left[X + \sqrt{X^2 - 1} \right],$$ (104)

where X is as defined in Equation (102) above.

5.4.2 Acceleration of the nth Ball from the Center to the Edge

Now, the ball is being accelerated from rest on the second track, where it will fly off the edge and travel back to the primary launcher. The orientation of this track is similar to the launching track on the primary launcher ($a = 1$). However, that track should be oriented such that the ball is launched backward and not forward. There is a difference of π in the angular position of the ball at the exit with respect to its position on the primary launcher. The time it takes the ball to reach the edge is, as determined before, 0.5 ms, and it will be the same for all balls. The exit time can be calculated from Equation (57) in *Chapter 3*. The value of k would be chosen so that the value of the time obtained here is greater than the time it takes this nth ball to reach the center, calculated in the previous section, plus the 0.5 ms needed to reach the edge of the second track when accelerated from the center of the relay, as explained in the beginning of this paragraph. We will also note that we start here with a value of $k = 1$, instead of 0 as in the case of the primary launcher, to take care of the orientation of the track that has to insure a backward launch.

5.5 POSITION OF THE RELAY

We need to determine the exact position of the moving relay when the nth ball is about to impact it. After the $(n-1)$th ball leaves the relay, this latter would be moving at a velocity already determined (second section of this chapter). Therefore, the distance it travelled from the time the $(n-1)$th ball left it, until the nth ball arrives, can be easily determined. This relay position should be compared to the distance that the nth ball travelled, from the time it exit the primary launcher, to the catching time. If the distance travelled by the ball is larger, as we will see from the results of the program, a mechanism should be included to slow down the ball for exact timing. This is one of the purposes of the guidance system. As we will describe it in details later, all three directions of motion would be corrected by the aiming mechanism. Vertical adjustment will insure the catching of the ball at the desired position, ζ_i, and a longitudinal adjustment will insure a synchronization between the incoming ball and the catching time on the relay.

5.6 ROTATIONAL ENERGY OF THE RELAY

The last part of this kinematics and dynamics study, would be to calculate the change in rotational energy the relay experiences due to its impact with all these little projectiles. For the first ball, caught and launched from the edge, there is no change in rotation energy, as explained in *Chapter 4*. However, for the nth ball, caught at a position between the edge and the center of the relay, and then accelerated all the way to the edge, the relay would lose some rotational energy. Let us determine exactly how much energy it would lose.

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We have:

$$\vec{v}_{in} = \begin{pmatrix} v_{exit} \\ 0 \end{pmatrix},$$

and

$$\vec{v}_{out} = \begin{pmatrix} -v_{exit} \\ 0 \end{pmatrix}.$$

We also have:

$$\vec{r}_{in} = \begin{pmatrix} r_i \cos(\omega t_i + \zeta_i) \\ r_i \sin(\omega t_i + \zeta_i) \end{pmatrix},$$

and,

$$\vec{r}_{out} = \begin{pmatrix} r_1 \cos(\omega t + \zeta_1) \\ r_1 \sin(\omega t + \zeta_1) \end{pmatrix},$$

where t is the time the nth ball flies off the edge of the relay and all other quantities are as defined previously.

Therefore, we can write the angular momentum as:

$$L_{in} = -mr_i v_{in} \sin(\omega t_i + \zeta_i),$$

and,

$$L_{out} = mr_1 v_{in} \sin(\omega t + \zeta_1).$$

These two quantities can be easily calculated and the difference, $L_{in} - L_{out}$, which is negative, represents the decrease in rotational energy after the nth ball impacts and leaves the relay. We have to compile all the decreases of energy, from the second to the last ball, to get the total lost of rotational energy. We should then be able to deduce the change in angular speed of the

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relay. We expect this change to be negligible, but we will calculate it in our program.

We are now ready to write the general program that codes all the different steps described in this chapter. We will mention though that we added one more thing at the beginning of this program. It will check first the sliding tendency based on the roll versus skid analysis in *Chapter 3*. If the ball is expected to slide on the track, it will ask the user to change the system parameters. Otherwise, it will inform him that the choice of parameters is fine and that the ball will roll nicely on the track. It will then go on calculating the data needed to summarize all the derivations and visualize the propulsion of the spaceborne relay.

We will include in this section a summary of the results for the first 20 balls launched. Table - 4 will give the timing information, and Table - 5 the positions information. The general code written is given in the appendix.

$t_{release}$ (ms)	t_{exit} (ms)	t_{catch} (ms)	t_{brtup} (ms)	t_{exrel} (ms)
0.404	0.905	294.719	0.501	295.901
1.661	2.162	296.604	0.402	297.785
2.917	3.418	297.860	0.374	299.042
4.174	4.675	299.117	0.355	300.299
5.431	5.932	300.373	0.341	301.555
6.687	7.188	301.630	0.329	302.812
7.944	8.445	302.886	0.319	304.069
9.201	9.701	304.142	0.311	305.325
10.457	10.958	305.399	0.303	306.582
11.714	12.215	306.655	0.296	307.839
12.970	13.471	307.912	0.290	309.095
14.227	14.728	309.168	0.284	310.352
15.484	15.985	310.425	0.279	311.609
16.740	17.241	311.681	0.274	312.865
17.997	18.498	312.938	0.270	314.122
19.254	19.755	314.194	0.265	315.378
20.510	21.011	315.451	0.261	316.635
21.767	22.268	316.707	0.257	317.892
23.024	23.524	317.964	0.254	319.148
24.280	24.781	319.220	0.250	320.405

Table - 4. Timing information

$t_{release}$: time at which the ball is released near the center of the primary launcher.

t_{exit}: time at which the ball flies off the launcher.

t_{catch}: time at which the ball is caught on the relay.

t_{brtup}: time at which the ball reaches the center of the relay.

t_{exrel}: time at which the ball flies off the relay.

ζ_i (rad.)	V_{relay} (m/s)	X_{relay} (m)	X_{ball} (m)
-2.303	1.361	500.035	500.035
-2.301	2.721	500.036	501.104
-2.300	4.080	500.036	501.103
-2.298	5.437	500.036	501.103
-2.297	6.794	500.037	501.103
-2.295	8.149	500.037	501.103
-2.294	9.504	500.038	501.102
-2.292	10.857	500.038	501.102
-2.291	12.209	500.039	501.102
-2.289	13.561	500.040	501.101
-2.288	14.911	500.041	501.101
-2.287	16.260	500.042	501.101
-2.285	17.608	500.043	501.100
-2.284	18.955	500.045	501.100
-2.282	20.300	500.046	501.100
-2.281	21.645	500.047	501.099
-2.279	22.989	500.049	501.099
-2.278	24.331	500.051	501.099
-2.276	25.673	500.052	501.099
-2.275	27.013	500.054	501.098

Table - 5. Positions information

ζ_i: catching position on the relay.
V_{relay}: velocity of the relay.
X_{relay}: position of the relay at catching time.
X_{ball}: position of the ball at catching time.

APPENDIX

COMPUTERIZED COMMUNICATION AND CONTROL SYSTEM FOR SPACEBORNE CENTRIFUGAL RELAYS FOR SPACECRAFT PROPULSION

One of the most important engineering problems for this project consists of accurately tracking the position of the projectiles and rotors and guiding each projectile exactly onto the appropriate guide tracks on each rotor. We have to control velocity vectors of the rapidly moving balls so that they encounter guide tracks on the spinning rotors at just the right location and time. We need to correct errors in lateral positions and make longitudinal adjustments to insure that projectiles arrive at the next relay in proper synchrony with its rotation.

Guidance studies include the tracking mechanism (interrupt of optical beam) and the aiming mechanism (electromagnetic focussing including the electromagnetic deflection coils design and the switching circuitry). A schematic of the system is given in figure 1.
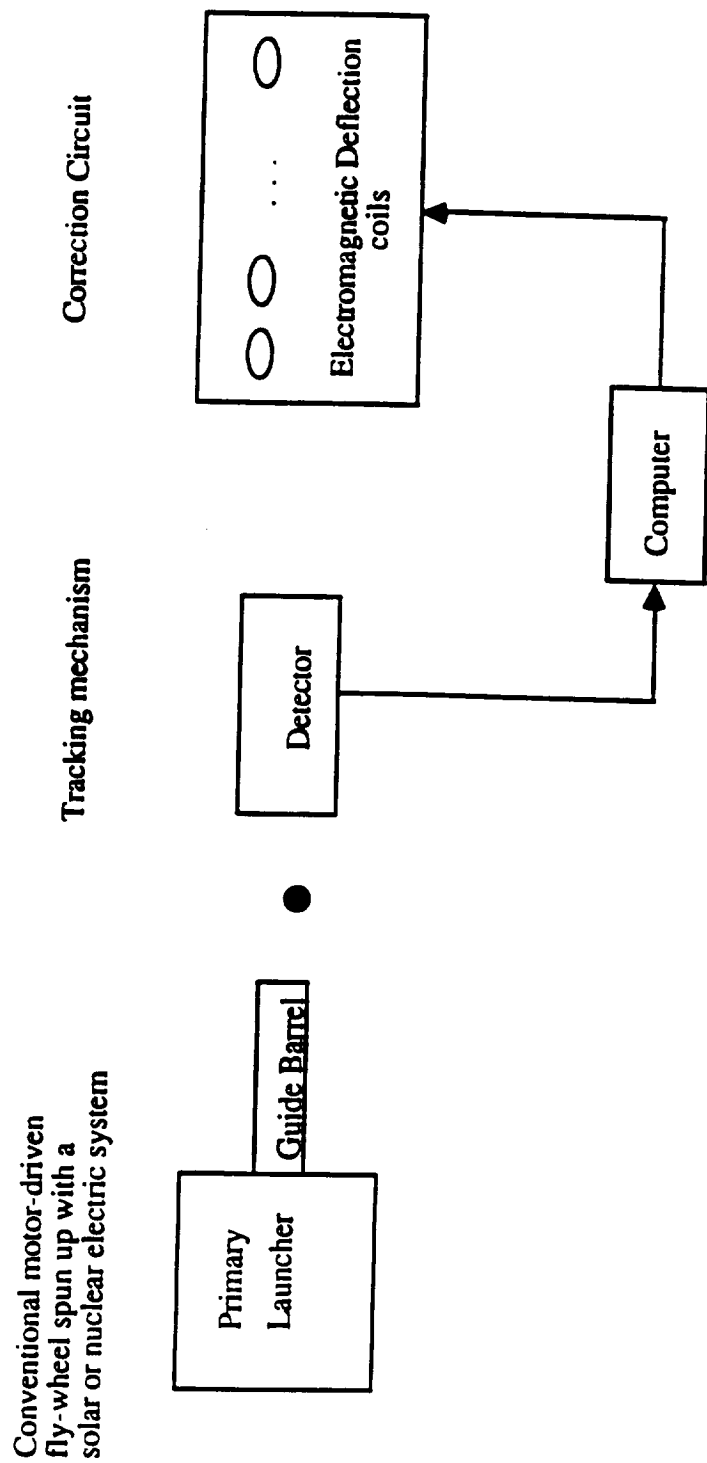
Figure-1 Computerized Communication and Control System

TRACKING MECHANISM

Interrupt of optical beam has been identified as the tracking mechanism. A narrow parallel laser beam is substantially diverged by a cylindrical lens with extremely short focal length. The diameter of the cylindrical lens has to be sufficiently small, but slightly greater than the nominal width of the raw laser beam. The lens should be about 1 to 2cm long and 2 to 4mm in diameter. We will also use an array of photodiodes to detect the position of the ball. Parallel output arrays of the type PDA-20 will be used. The precision on the position of the rapidly moving ball is of the order of 10^{-5} m.

We will be using two beam-photodiode array systems to determine both lateral positions of the ball. Then a similar system down the track will give the position of the ball at a later time allowing us to determine the drift in lateral position and also a measure of the longitudinal speed.

The mechanism is illustrated in figures 2 and 3.

A second method has also been investigated although the interrupt beam remains the leading candidate tracking mechanism. This potential method consists of creating a uniform electric field that will be perturbed by the presence of the conducting sphere. A grid will be installed along the field and a measure of the potential on different points of this grid will give us the position of the sphere. This principle is illustrated in figure 4.

Laser Cylindrical lens F Umbra d Photodiode array (n diodes)
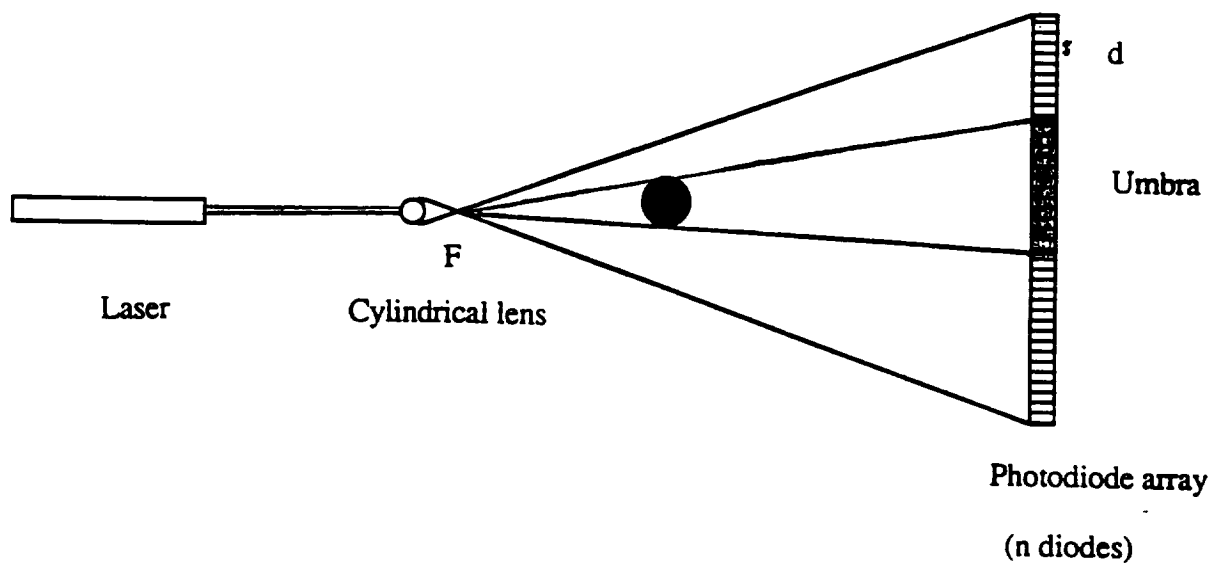
Figure-2. Interrupt Beam
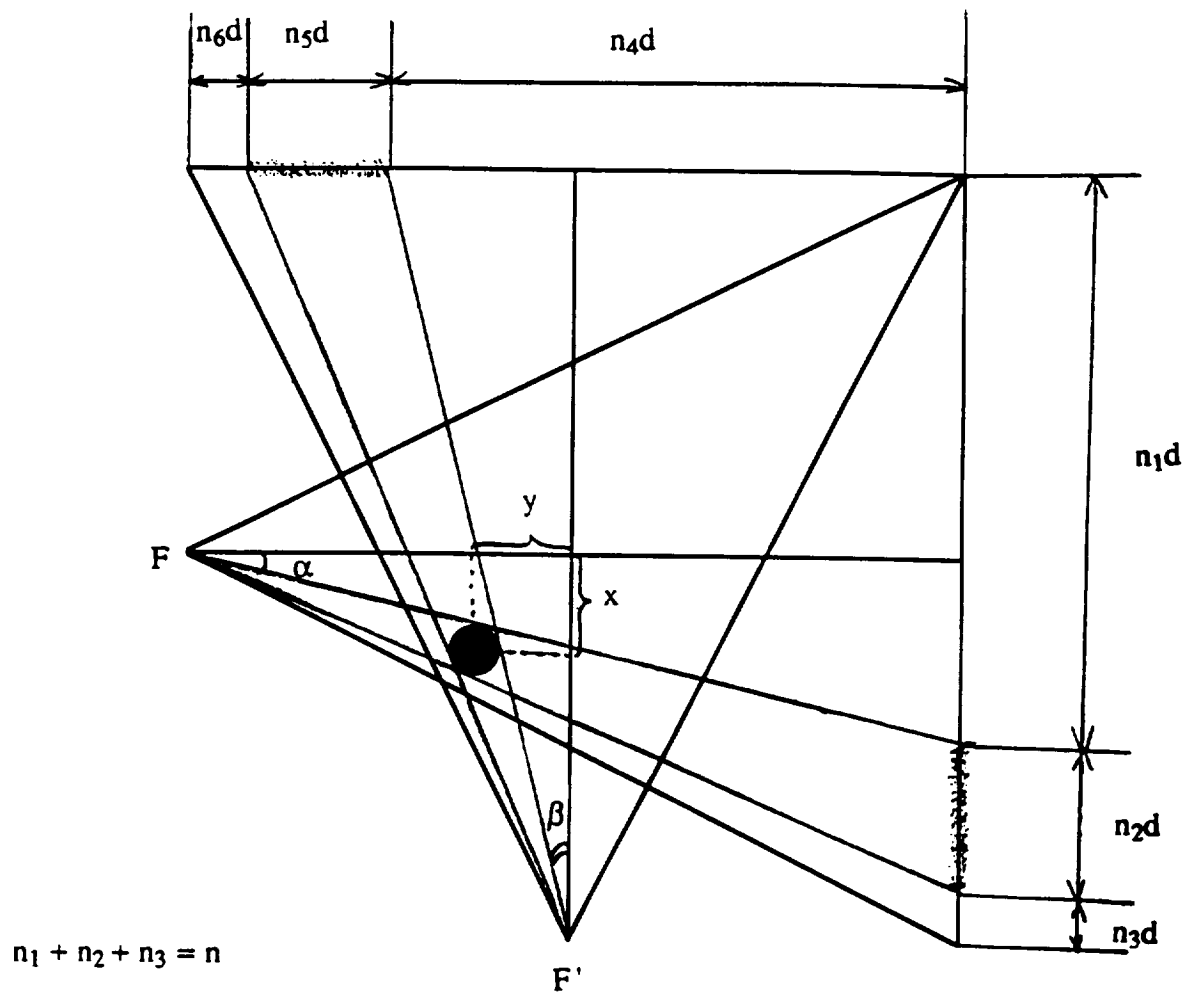
$n_1 + n_2 + n_3 = n$

$n_4 + n_5 + n_6 = n'$

$$tg\ \alpha = \frac{\frac{n}{2} - n_2 - n_3}{n'} \qquad\qquad tg\ \beta = \frac{\frac{n'}{2} - n_5 - n_6}{n}$$

$$x = r + \frac{\frac{d}{2}(n' - n\ tg\beta)\ tg\alpha - r\ tg\alpha\ (1 - tg\beta)}{1 - tg\alpha\ tg\beta}$$

$$y = \frac{n'}{2}\ d - \frac{\frac{d}{2}(n' - n\ tg\beta) - r\ (1 - tg\beta)}{1 - tg\alpha\ tg\beta}$$

r = radius of the sphere.

Figure-3. Interrupt Beam For Both Lateral Positions
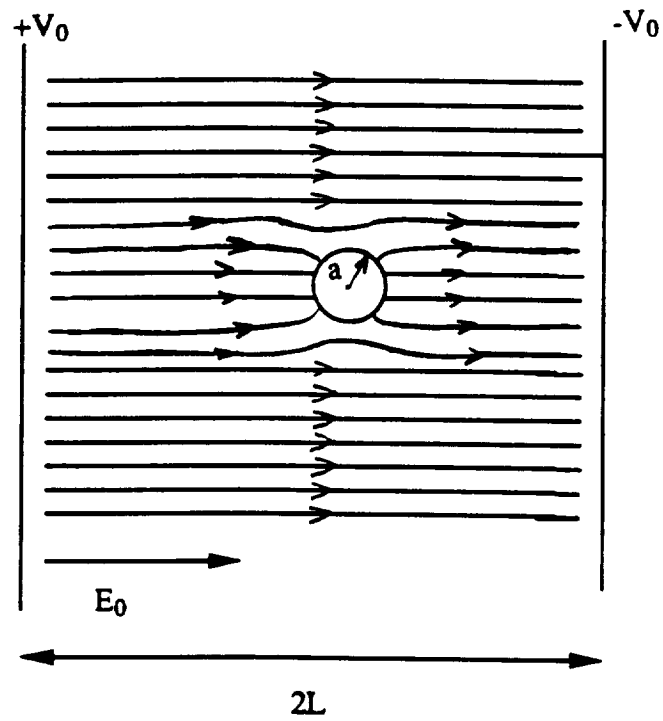
$$V(r,\theta) = -E_0 \ (1 - (a/r)^3 \ ) \ r \cos\theta$$

Figure-4. Conducting Sphere in a Uniform Electric Field

AIMING MECHANISM

For the velocity trimming, we propose that the projectiles be directed through a guide barrel after release from the centrifugal accelerators to limit the dispersion of lateral velocities. We identified the concept of electromagnetic focussing as the final aiming mechanism. When the ball arrives to the correction circuit it will be magnetized. Its magnetic moment and the external magnetic field that we create will produce a potential energy. A force will then be acting on the sphere modifying its position. The velocity errors are expected to vary randomly in magnitude and in direction from one projectile to the next one launched. We need the switching time of the electromagnetic circuit to be very small in order for us to set the necessary amount of magnetic induction in the circuit to make the appropriate correction for each projectile. The first thing we need to determine is the diffusion time of the magnetic field inside the sphere to make sure that it will have enough time to get magnetized while passing through the electromagnetic deflection coil at a high velocity. For this, we solved Maxwell equations,

$$\nabla \times H = \frac{4\pi}{c} J + \frac{1}{c} \frac{\partial D}{\partial t}$$
$$\nabla \times E + \frac{1}{c} \frac{\partial B}{\partial t} = 0$$
$$\nabla \cdot D = 4\pi\rho$$
$$\nabla \cdot B = 0$$

and got the diffusion equation,

$$\frac{\partial B}{\partial t} = \nabla \times (v \times B) + \frac{\mu c^2}{4\pi\sigma} \nabla^2 B$$

The magnetic field decays away exponatially in a diffusion time:

$$\tau = \frac{4\pi\sigma L^2}{\mu c}$$

For a steel ball of 1cm radius, $\tau = 0.6\mu S$ meaning that the magnetization can be considered as instantaneous compared to the total time spent by the sphere on one set of coils ($20\mu S$) and no premagnetization is necessary.

The magnetization of the ball can be expressed as:

$$M = \frac{3}{4\pi} \frac{\mu-1}{\mu+2} B = \frac{m}{V}$$

where: M = magnetization

m = magnetic moment

V = volume of the sphere

μ = permeability of the material (Steel)

B = magnetic induction of the external field

and the potential energy obtained is:

$$U = -m \cdot B$$

giving a force:

$$F = \rho V \gamma = -\nabla . U$$

where: ρ = volumic mass

γ = accceleration

$$\gamma = \frac{3}{4\pi\rho} \frac{\mu-1}{\mu+2} \nabla.B^2$$

This acceleration will compensate the error on the velocity:

$$V = V_0 - n\gamma t = 0$$

V_0 = initial error (about 0.1m/s)

n = number of loops

The magnetic induction producing this acceleration is obtained when a current I is circulating in a loop of radius a:

$$B = \mu_0 \frac{Ia^2}{2(a^2 + z^2)^{3/2}} \quad \text{(along the z-axis)}$$

$$\nabla.B^2 = -\frac{3}{2} \mu_0^2 I^2 a^4 \frac{z}{(a^2 + z^2)^4}$$

Using N turns instead of one in every loop will make the current needed to produce the same magnetic induction N times smaller. The total resistance R of the coils (for one loop) is given by:

$$R = Ro \frac{l}{s}$$

where: Ro = resistivity of the wire (copper)

l = total length of the wire

s = cross-section of the wire

The voltage necessary to produce the required current is:

$$V = RI$$

We have to minimize the power losses:

$$P = RI^2$$

the inductance L of the coils,

$$L = \mu_0 \frac{N^2}{h} \pi \, a^2 \, K$$

where   h = height of the short solenoid

a = radius of the winding

K = coefficient

and the time constant,

$$\tau = \frac{L}{R}$$

We explored the tradeoff between switching speed and power losses with an aim to find the optimum compromise between these two parameters. An acceptable switching scheme has been found using the previous equations. For a projectile (1cm radius steel ball) launched at 1km/s and arriving at the correction circuit with an error of 0.1m/s the characteristics of the deflection coils are as follows. Using 10 identical sets of coils (2cm in diameter for each set), the magnetic induction necessary for the compensation of the velocity error is such that $\nabla.B^2 = 1.65$ T^2/m, the corresponding current is 10.1A assuming 100

turns in each set of coils. The resistance for one set of coils is 0.4Ω, the diameter of the wire is 0.5mm, the inductance is 0.18mH and the time constant is 0.45ms. The voltage needed is 4.04V and the power losses are 40.8W corresponding to an increase in the temperature of 19°K for one second. These studies indicate that electromagnetic deflection is feasible for the magnitude of velocity errors one normally obtains with a rifle barrel.

A representation of the coil and the switching circuitry is given in figure 6. The switches used are silicon controlled rectifiers, and the capacitors are used for faster response. The coil and its power supply are illustrated in figure 5.
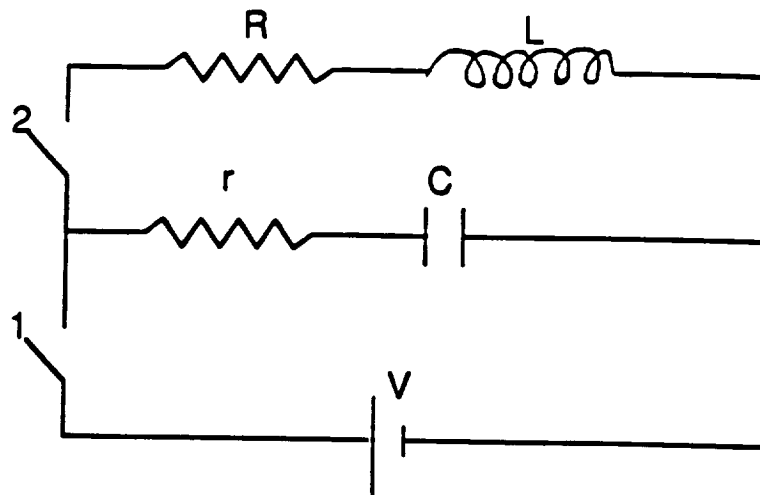
1- Charge

$$V_c = V[1-\exp(-t/rc)]$$
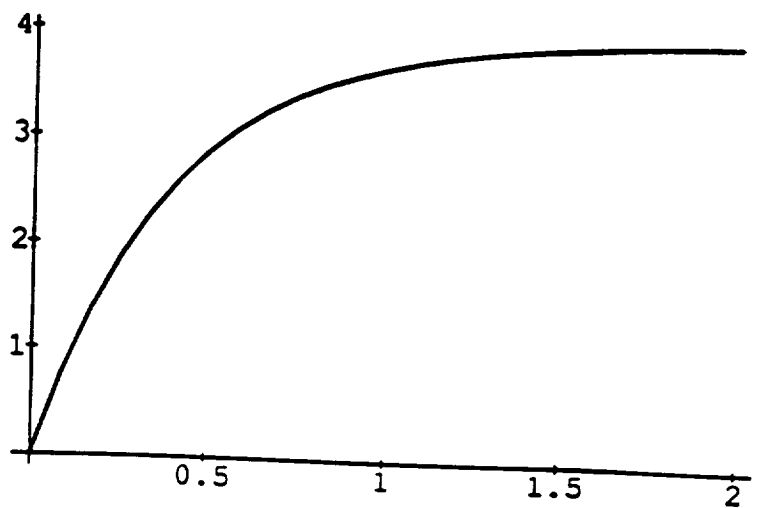
2-

$$(r + R)^2 = 4L/C$$

$$i = \frac{V}{R + r} \left[1 - \frac{R + r}{2L} \, t\right] \exp\left(- \frac{R + r}{2L} \, t\right)$$

Minimize rc , r=R

$$\begin{cases} r=R=0.4 \ \Omega \\ L=0.18 \ mH \\ C=1 \ mF \end{cases}$$

Figure-5. Coil and Power Supply
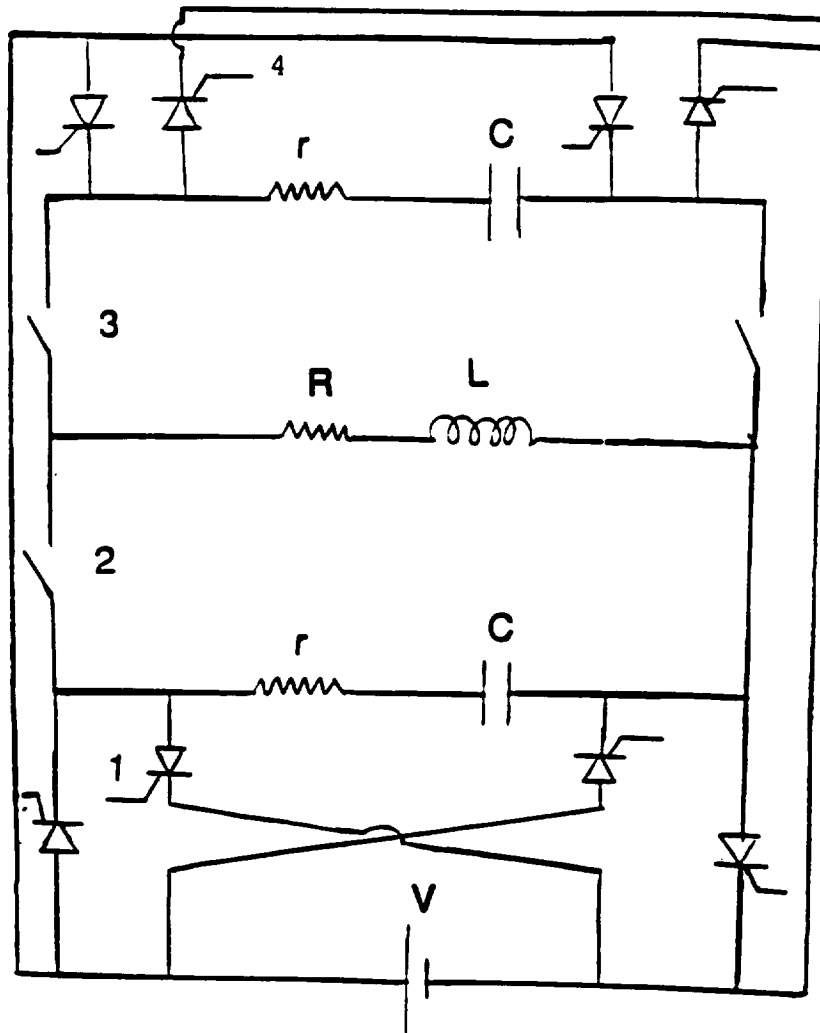
Figure-6. Switching Circuitry

APPENDIX

We will present in this section the computer programs written (in c- language), to generate the data needed for understanding the behavior of the system as a function of certain parameters. We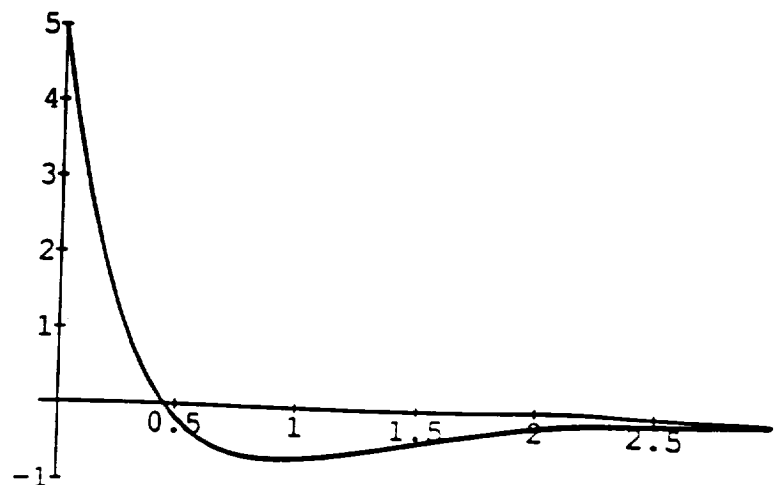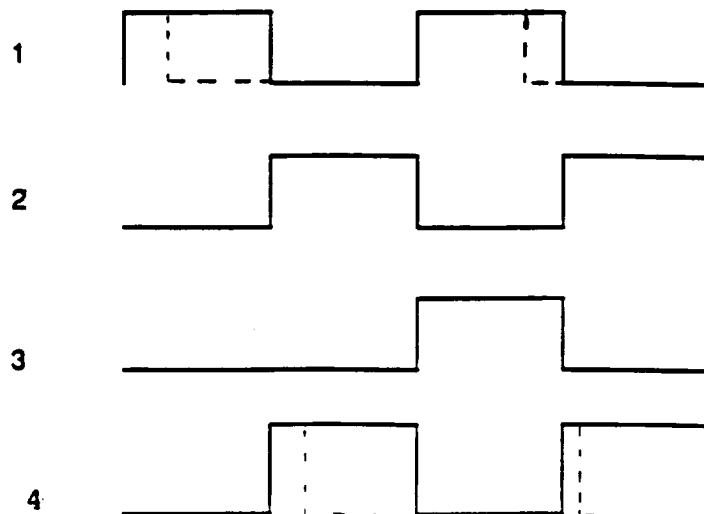 will start with RELAYVE-LOCITY.C, the velocity of the relay versus the number of balls launched, as illustrated in Figure-5 in the 2nd chapter. The second program we wrote, EXITVELOCITY.C, illustrates the dependency of the exit velocity of the balls from the primary launcher, V_{exit}, on the track shape constant, a. An illustration of V_{exit}/V_{tip} was given in Figure - 13. The third program written, ROLLSKIDW.C, represents the sliding tendency of the ball on the track as a function of the angular velocity ω. An illustration of the ratio λ_2/λ_1, versus ω, for four different values of the track shape constant, a, was given in Figure - 17. The fourth program, ROLLSKIDA.C, represents the maximized sliding tendency of the ball on the track as a function of the track shape constant a. An illustration of the ratio $(\lambda_2/\lambda_1)_{max}$, versus a, was given in Figure - 18. Finally, the general code that summarizes all the analysis done in *Chapter 2* through *Chapter 5*, KIN&DYN.C, is given in this last part of the appendix. This code generates all the data necessary to visualize the concept of acceleration of a spaceborne payload. This program can be run for an arbitrary number of balls, but the number n was chosen equal to 20. The data obtained was given in Table - 4 and Table - 5.

```
/* RELAYVELOCITY.C */


/*  THIS PROGRAM CALCULATES THE RELAY VELOCITY  */


#include "stdio.h"
main ()
{
FILE *fp;
float V_exit=1701; /* this value of the exit velocity of the ball was
                      calculated in the numerical application in the
                      second chapter */
float M=10;         /* mass of the relay in kg */
float m=4e-3;       /* mass of the ball in kg */
float eps=m/M;
float V[8010];
int i;
int n=8000;         /* number of balls launched */
V[0]=0;             /* relay initial velocity */


             /* Opening of a file for the data */


if((fp=fopen("velocity.dat","w"))==NULL)
  {
   printf("cannot open file/n");
   exit(1);
  }
```

```
                    /* Calculation of the relay velocity */

for(i=0;i<n;i++)
   {
    V[i+1]=V[i]+2*eps*(V_exit-V[i]);
    fprintf(fp,"%f\n",V[i]);
    }
}
```

```c
/* EXITVELOCITY.C */

/* THIS PROGRAM ILLUSTRATES THE RELATIONSHIP BETWEEN
   THE  EXIT VELOCITY OF THE BALLS FROM THE PRIMARY
   LAUNCHER, AND THE TRACK SHAPE CONSTANT. */

#include "stdio.h"
#include <math.h>
main()
{
FILE *fp;
float r0=1e-2;      /* distance from center of rotor to beginning
                       of track (in meters) */
float r1=10e-2;     /* radius of the rotor (in meters) */
float omega=10e+3;  /* rotor angular speed (in radians per second) */
float a=0.5;        /* track shape constant */
float V_tip;        /* rotor tip speed (in meters per second) */
float V_exit;       /* exit velocity of the balls from the primary
                       launcher */
int i;
float ratio[15];

                /* Opening of a file for the data */

if((fp=fopen("exitvelocity.dat","w"))==NULL)
  {
   printf("cannot open file\n");
```

```
    exit(1);
  }


            /* Calculation of the exit velocity of the ball */

V_tip=omega*r1;
for(i=0;i<=10;i++)
{
 V_exit=V_tip*sqrt(1+(5./7)*(1-(r0/r1)*(r0/r1))+2*sqrt(5./7)
        *sqrt(1-(r0/r1)*(r0/r1))*1/sqrt(1+a*i*a*i));
 ratio[i]=V_exit/V_tip;
 fprintf(fp,"%f   %f\n",a*i,ratio[i]);
 }
}
```

```
                        /* ROLLSKIDW.C */


                 /* THIS IS AN ANALYSIS OF THE SLIDING TENDENCY
                    OF THE BALL ON THE TRACK AS A FUNCTION OF
                    THE ANGULAR VELOCITY OF THE ROTOR */


    #include "stdio.h"
    #include <math.h>
    main()
    {
    FILE *fp;
    float r1=10e-2;   /* radius of the rotor */
    float omega;      /* rotor angular speed (in radians per second) */
    float omega0=1000.0;
    float a;          /* track shape constant */
    float V_tip;      /* rotor tip speed (in meters per second) */
    float K0;
    float K;          /* constant appearing in the equations of motion */
    float t=0.5e-3;   /* time at which ball reaches the edge
                          of the track */
    float th;
    float ratio1[25];/* required coefficient of friction */
    float ratio2[25];
    float ratio3[25];
    float ratio4[25];
    int i;
```

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```
                    /* Opening of a file for the data */


if((fp=fopen("rollskidw.dat","w"))==NULL)
  {
   printf("cannot open file\n");
   exit(1);
  }


                /* Calculations and writing of the data */

V_tip=omega*r1;
a=0.5;
for(i=1;i<=20;i++)
   {
    omega=omega0*i;
    K0=5./(7*(1+1/(a*a)));
    K=omega*sqrt(K0);
    th=tanh(K*t);
    ratio1[i]=2*a*sqrt(1+a*a)/(5*(1+th*th)+2*sqrt(35*(1+a*a))*th);
   }
a=1.;
for(i=1;i<=20;i++)
   {
    omega=omega0*i;
    K0=5./(7*(1+1/(a*a)));
    K=omega*sqrt(K0);
    th=tanh(K*t);
```

```
        ratio2[i]=2*a*sqrt(1+a*a)/(5*(1+th*th)+2*sqrt(35*(1+a*a))*th);
    }
a=3.;
for(i=1;i<=20;i++)
    {
      omega=omega0*i;
      K0=5./(7*(1+1/(a*a)));
      K=omega*sqrt(K0);
      th=tanh(K*t);
      ratio3[i]=2*a*sqrt(1+a*a)/(5*(1+th*th)+2*sqrt(35*(1+a*a))*th);
    }
a=10.;
for(i=1;i<=20;i++)
    {
      omega=omega0*i;
      K0=5./(7*(1+1/(a*a)));
      K=omega*sqrt(K0);
      th=tanh(K*t);
      ratio4[i]=2*a*sqrt(1+a*a)/(5*(1+th*th)+2*sqrt(35*(1+a*a))*th);
      fprintf(fp,"%f %f %f %f %f\n",omega,ratio1[i],ratio2[i],
              ratio3[i],ratio4[i]);
    }
}
```

```
                    /* ROLLSKIDA.C */


              /* THIS IS AN ANALYSIS OF THE SLIDING TENDENCY
                 OF THE BALL ON THE TRACK AS A FUNCTION OF
                 THE TRACK SHAPE CONSTANT */


#include "stdio.h"
#include <math.h>
main()
{
FILE *fp;
float r1=10e-2;         /* radius of the rotor */
float omega=10e+3;      /* angular velocity of the rotor */
float a;                /* track shape constant */
float a0=0.5;
float V_tip=omega*r1;
float ratio[25];        /* required coefficient of friction */
int i;


              /* Opening of a file for the data */


if((fp=fopen("rollskida.dat","w"))==NULL)
  {
   printf("cannot open file\n");
   exit(1);
  }
```

```
                /* Calculations and writing of the data */

for(i=0;i<=20;i++)
   {
    a=a0*i;
    ratio[i]=2*a*sqrt(1+a*a)/5;
    fprintf(fp,"%f %f\n",a,ratio[i]);
   }
}
```

```c
/* KIN&DYN.C */


/* KINEMATICS AND DYNAMICS OF THE PRIMARY LAUNCHER,
     THE RELAY, AND THE PROJECTILES. */


#include "stdio.h"
#include <math.h>
main()
{
FILE *fp1;
FILE *fp2;


        /* DEFINITION OF THE PARAMETERS OF THE SYSTEM
         (All distances are in meters and masses in kilograms) */


float r0=1e-2;          /* distance from the center of the rotor to
                            the beginning of the track */
float r1=10e-2;         /* radius of the rotor */
float l=0.5e-2;         /* radius of the ball */
float M=10;             /* mass of the rotor */
float m=4e-3;           /* mass of the ball */
float omega=10e+3;      /* rotor angular speed (in radians per second) */
float a=1;              /* track shape constant */
float MU_avai=0.93;     /* available coefficient of friction; in this case,
                            "stainless steel-stainless steel" */
float a_max;            /* maximum track shape constant allowed to
                            avoid skidding */
```

```
float MU_req;          /* maximum coefficient of friction required
                          to avoid skidding */
float epsilon=m/M;     /* mass ratio */
float pi=3.141592654;
float T=2*pi/omega;    /* rotor period of rotation (in seconds) */
float V_tip=omega*r1;/* rotor tip speed (in meters per second) */
float V_exit;          /* exit velocity of the balls from the
                          primary launcher */
float K0=5./(7*(1+1/(a*a)));
float K;               /* constant appearing in the equations
                          of motion */
int n=20;              /* number of balls to be launched */
float t_exit[25];      /* t_exit[n]=time at which the nth ball
                          flies off the launcher */
float t_release[25];   /* t_release[n]=time at which the nth ball
                          is released near the center of the launcher */
float t_brt;           /* time it takes the ball to roll on the
                          track From r0 to r1 */
float V_relay[25];     /* V_relay[n]=relay linear velocity just as
                          the nth ball leaves the relay heading back
                          toward the launcher */
float t_catch[25];     /* t_catch[n]=time at which the nth ball is
                          caught on the track of the relay */
float zeta_c[25];      /* zeta_c[n]=angular position of the nth ball on
                          the track at catching time */
float tg;
float r_c;             /* r_c=radial position of the ball on the track
```

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```
                                    at catching time */
float rdot_c;           /* rdot_c=d(r_c)/dt */
float U;                /* rotational speed of the ball (at the tip) */
int k;
int i;
float t_flight;         /* time of flight of ball between the launcher
                           and the relay */
float d_min;            /* minimal initial separation between
                           the launcher and the relay */
float d;                /* actual initial separation */
float Del;
float X1;
float X2;
float X;
float t;
float t_brtup[25];      /* t_brtup[n]=time for the nth ball to roll up
                           the track and reach the center of the relay
                           when starting at the impact point */
float t_exrel[25];      /* t_exrel[n]=time at which the nth ball
                           exits the relay and travels back to the
                           primary launcher */
float X_relay[25];      /* X_relay[n]=position of the relay when it
                           is about to catch the nth ball */
float X_ball[25];       /* X_ball[n]=position of the ball at
                           catching time */
V_relay[0]=0.;          /* relay initial velocity */
K=omega*sqrt(K0);
```

```c
/* ROLL VERSUS SKID ANALYSIS */

a_max=sqrt((sqrt(1+25*MU_avai*MU_avai)-1)/2);
MU_req=2*a*sqrt(1+a*a)/5;
if(MU_avai>MU_req)
   {
   printf("Ball will roll nicely on track.  Your choice of system
           parameters is fine.You could choose 'a' as high as %f
           if you wanted to).\n",a_max);
   }
else
   {
   printf("Ball will skid on track.  Choose another pair
           of materials to change the value of MU_avai or,
           simply choose a smaller value for 'a'. The maximal
           acceptable value for 'a' is %f\n",a_max);
           exit();
   }


       /* EXIT VELOCITY OF THE BALLS FROM THE PRIMARY LAUNCHER */

printf("The rotor tip speed is equal to %f m/s. \n",V_tip);
V_exit=V_tip*sqrt(1+(5./7)*(1-(r0/r1)*(r0/r1))+
       2*sqrt(5./7)*sqrt(1-(r0/r1)*(r0/r1))*1/sqrt(1+a*a));
printf("The speed of the ball when it flies off the track of the
           primary launcher is equal to %f m/s. \n",V_exit);
```

```
/* FORWARD LAUNCH (PRIMARY LAUNCHER --- RELAY) */

    /* Computation of the time it takes the ball to reach
        the edge of the track (distance r1) when released
        near the center at a distance r0 */

t_brt=(1./K)*log(r1/r0+sqrt((r1/r0)*(r1/r0)-1));
printf("The time it takes the ball to reach the edge of the
        track, when released near the center of the rotor
        is texit[1]-trelease[1] = %f s.\n",t_brt);

    /* Computation of the time at which the 1st ball exits
        the launcher */

k=0;
label1:t_exit[1]=(1./omega)*(atan((-1/a)-(V_tip/(K*sqrt((r1*r1)-
        (r0*r0))))))-(1./a)*log(r1/r0)+k*pi);
while(t_exit[1]<t_brt)
    {
    k=k+2;
    goto label1;
    }

    /* Computation of the time at which the 1st ball
        is released near the center of the launcher */
```

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```
t_release[1]=t_exit[1]-t_brt;


              /* Times at which the ith ball is released and
                 exits the launcher, assuming we are launching
                 one ball every 2 revolutions of the rotor */


for(i=1;i<=n;i++)
   {
   t_exit[i]=t_exit[1]+2*(i-1)*T;
   t_release[i]=t_release[1]+2*(i-1)*T;
   }


              /* RELAY VELOCITY AFTER THE LAUNCH OF THE ith ball */


for(i=1;i<=n;i++)
   {
   V_relay[i]=V_relay[i-1]+2*(epsilon/(1+epsilon))*(V_exit-V_relay[i-1]);
   }


              /* DETERMINATION OF THE CATCHING POSITION AND TIME */


              /* Catching position and time for the 1st ball */

U=sqrt(5./7)*omega*sqrt((r1*r1)-(r0*r0));
tg=sqrt(pow((V_exit-V_relay[0])/U,2)*(1+1/pow(a,2))-1);
zeta_c[1]=-(1/a)*log((-U*(1-a*tg))/(omega*r0*sqrt(1+pow(a,2))));
k=1;
```

```
label2:t_catch[1]=(1/omega)*(atan(tg)-zeta_c[1]+k*pi);
while(t_catch[1]<=t_exit[1])
    {
    k=k+2;
    goto label2;
    }


        /* Initial separation between the primary launcher
            and the relay */


t_flight=t_catch[1]-t_exit[1];
d_min=V_exit*t_flight;
printf("The minimal initial separation is %f m. \n",d_min);
printf("It is better to start with a distance of at least
        500 m, which means that the ball is caught several
        revolutions later than this value of t_catch[1]
        determined above. Let us add then to t_catch[1],
        2k(pi)/omega until we reach our goal of 500 m.\n");
k=1;
label3:t_catch[1]=(1/omega)*(atan(tg)-zeta_c[1]+k*pi);
t_flight=t_catch[1]-t_exit[1];
d=V_exit*t_flight;
while(d<500)
    {
    k=k+2;
    goto label3;
    }
```

```c
printf("The initial separation is now set to %fm.\n",d);

            /* Catching position and time for the ith ball */

for(i=2;i<=n;i++)
   {
   tg=sqrt(pow((V_exit-V_relay[i-1])/U,2)*(1+1/pow(a,2))-1);
   if(tg>1.)
     {
     zeta_c[i]=-(1/a)*log((-U*(1-a*tg))/(omega*r0
             *sqrt(1+pow(a,2))));
     label4:t_catch[i]=(1/omega)*(atan(tg)-zeta_c[i]+k*pi);
     while(t_catch[i]<=(t_exit[i]+t_flight))
         {
         k=k+2;
         goto label4;
         }
     }
   else
      {
      printf("The maximum number of balls that can be caught
             is %d\n",i-1);
      exit();
      }
   }


                        /* BALL ON RELAY */
```

```
                    /* 1st phase: from impact point to center of relay */


for(i=1;i<=n;i++)
   {
   r_c=r0*exp(-a*zeta_c[i]);
   rdot_c=(a*r_c*omega-a*sqrt(pow(V_exit,2)*(1+a*a)-
           pow(a*r_c*omega,2)))/(1+a*a);
   Del=(pow((rdot_c/K),2))*(pow((rdot_c/K),2)-(r_c*r_c-r0*r0));
   X1=((-r_c*r0)-sqrt(Del))/((pow((rdot_c/K),2))-(r_c*r_c));
   X2=((-r_c*r0)+sqrt(Del))/((pow((rdot_c/K),2))-(r_c*r_c));
   if(X1>0)
     {
     if(X1<=10)
       {
       X=X1;
       }
     else
        {
        X=X2;
        }
     }
   else
      {
      X=X2;
      }
```

```
t_brtup[i]=(1./K)*(log(X+sqrt((X*X)-1.0)));
}


        /* 2nd phase: from center of relay to edge of track */


for(i=1;i<=n;i++)
   {
    k=1;
    label5:t_exrel[i]=(1./omega)*(atan((-1/a)-(V_tip/(K*sqrt((r1*r1)
                    -(r0*r0))))))-(1./a)*log(r1/r0)+k*pi);
    t=t_catch[i]+t_brtup[i]+t_brt;
    while(t_exrel[i]<t)
         {
          k=k+2;
          goto label5;
         }
   }


                        /* POSITION OF THE RELAY */


X_relay[1]=d;
for(i=2;i<=n;i++)
   {
   X_relay[i]=X_relay[i-1]+(V_relay[i-1]*(t_catch[i]-t_exrel[i-1]));
   }


        /* POSITION OF THE ITH BALL AT THE TIME OF CATCH */
```

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```c
X_ball[1]=d;
for(i=2;i<=n;i++)
   {
   X_ball[i]=V_exit*(t_catch[i]-t_exit[i]);
   }


                    /* OPENING OF FILES FOR THE DATA */


if((fp1=fopen("time.dat","w"))==NULL)
  {
  printf("cannot open file\n");
  exit(1);
  }
if((fp2=fopen("position.dat","w"))==NULL)
  {
  printf("cannot open file\n");
  exit(1);
  }


                    /* WRITING THE DATA */


for(i=1;i<=n;i++)
   {
   fprintf(fp1,"%f %f %f %f %f\n",t_release[i],t_exit[i],
           t_catch[i],t_brtup[i],t_exrel[i]);
   fprintf(fp2,"%f %f %f %f \n",zeta_c[i],V_relay[i],
```

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```
                    X_relay[i],X_ball[i]);
        }
}
```

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